Mathematics

10

Sample Question Paper

Standard (Code 041)

ANSWERS

Section - A

1.	(C)	2.	(<i>a</i>) 3. (<i>c</i>)
4.	(b)	5.	(<i>d</i>) 6. (<i>a</i>)
7.	(b)	8.	(<i>d</i>) 9. (<i>b</i>)
10.	(C)	11.	(c) 12. (a)
13.	(C)	14.	(<i>a</i>) 15. (<i>c</i>)
16.	(C)	17.	(<i>a</i>) 18. (<i>b</i>)
19.	(C)	20.	(b)

Section - B

21. We have,

$$\frac{3}{2}x - \frac{5}{3}y = -2$$

9x - 10y = -12 ...(1)
$$\frac{1}{3}x + \frac{1}{2}y = \frac{13}{6}$$

 \Rightarrow

 \Rightarrow

 \Rightarrow

2x + 3y = 13 ...(2)

Multiplying eqn (1) by 3 and eqn (2) by 10, we get

$$27x - 30y = -36 \qquad \dots (4)$$

$$20x + 30y = 130$$
 ...(5)

Adding equations (4) and (5), we get

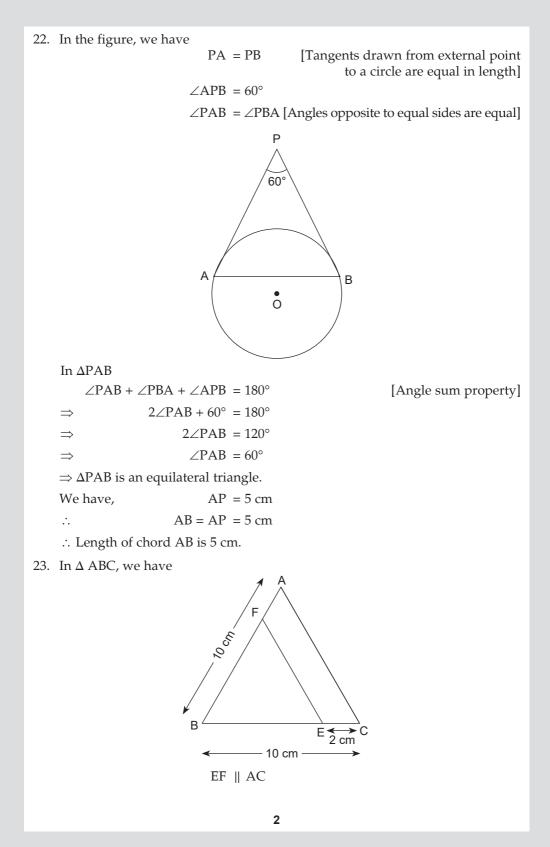
$$47x = 94$$
$$x = \frac{94}{47} = 2$$

$$\Rightarrow$$

 \Rightarrow

Putting value of x = 2 in equation (1), we get

$$9 \times 2 - 10y = -12$$
$$-10y = -30$$
$$y = 3$$



	BC = 10 cm, AB = 13 cm and EC = 2 cm	
	$\frac{BE}{EC} = \frac{BF}{AF}$	[By B.P.T.]
\Rightarrow	$\frac{BC - EC}{EC} = \frac{AB - AF}{AF}$	
\Rightarrow	$\frac{10-2}{2} = \frac{13-AF}{AF}$	
\Rightarrow	$\frac{8}{2} = \frac{13 - AF}{AF}$	
\Rightarrow	8AF = 2(13 - AF)	
$\begin{array}{c} \Rightarrow \\ \Rightarrow \\ \Rightarrow \end{array}$	8AF = 26 - 2AF	
\Rightarrow	10AF = 26	
\Rightarrow	AF = 2.6 cm	

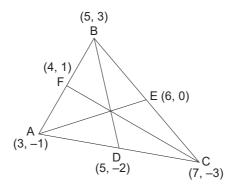
24. The coordinates of the vertices of the given $\triangle ABC$ are

A(
$$x_1 = 2, y_1 = 3$$
), B($x_2 = -2, y_2 = 1$) and C($x_3 = 3, y_3 = -2$)
Area of $\triangle ABC = \frac{1}{2} \left[\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right] \right]$
 $= \frac{1}{2} \left[\left[2(1+2) + (-2)(-2-3) + 3(3-1) \right] \right]$ sq units
 $= \frac{1}{2} \left[\left[2(3) - 2(-5) + 3(2) \right] \right]$
 $= \frac{1}{2} \left[\left[6 + 10 + 6 \right] \right]$
 $= \frac{1}{2} (22) = 11$ sq units

Hence, the area of the triangle is 11 sq units.

or

24. Let the points (3, –1), (5, 3) and (7, –3) be denoted by A, B and C respectively. Let D, E and F be the mid-points of AC, BC and BA respectively.



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The coordinates of D, E and F respectively are

$$D\left(\frac{3+7}{2}, \frac{-1-3}{2}\right), \ E\left(\frac{5+7}{2}, \frac{3-3}{2}\right) \text{ and } F\left(\frac{5+3}{2}, \frac{3-1}{2}\right)$$

i.e. D(5, -2), E(6, 0), F(4, 1)

:. The medians BD, CF and AE are given by

BD =
$$\sqrt{(5-5)^2 + (-2-3)^2} = \sqrt{0^2 + (-5)^2} = \sqrt{25} = 5$$
 units
CF = $\sqrt{(4-7)^2 + (1+3)^2} = \sqrt{(-3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$ units
AE = $\sqrt{(6-3)^2 + (0+1)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$ units

Hence, length of the medians are 5 units, 5 units and $\sqrt{10}$ units.

25. Given:
$$x = a \cos \theta - b \sin \theta$$
, $y = a \sin \theta + b \cos \theta$

$$x^{2} + y^{2} = (a \cos \theta - b \sin \theta)^{2} + (a \sin \theta + b \cos \theta)^{2}$$
$$= a^{2} \cos^{2} \theta + b^{2} \sin^{2} \theta - 2ab \sin \theta \cos \theta + a^{2} \sin^{2} \theta$$
$$+ b^{2} \cos^{2} \theta + 2ab \sin \theta \cos \theta$$
$$= a^{2} \cos^{2} \theta + b^{2} \sin^{2} \theta + a^{2} \sin^{2} \theta + b^{2} \cos \theta$$
$$= a^{2} (\cos^{2} \theta + \sin^{2} \theta) + b^{2} (\sin^{2} \theta + \cos^{2} \theta)$$
$$= a^{2} + b^{2} \qquad [\because \cos^{2} \theta + \sin^{2} \theta = 1]$$

Hence, proved.

or

25. Given: $\tan \theta = \frac{a}{b}$

L.H.S.
$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$$

Dividing numerator and denominator by $b \cos \theta$

$$\frac{a\sin\theta}{b\cos\theta} - \frac{b\cos\theta}{b\cos\theta} = \frac{a}{b}\tan\theta - 1$$

$$\frac{a\sin\theta}{b\cos\theta} + \frac{b\cos\theta}{b\cos\theta} = \frac{a}{b}\tan\theta + 1$$

$$= \frac{\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) - 1}{\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) + 1} = \frac{\frac{a^2}{b^2} - 1}{\frac{a^2}{b^2} + 1} \qquad \left[\because \tan\theta = \frac{a}{b}\right]$$

$$= \frac{a^2 - b^2}{a^2 + b^2} = \text{R.H.S}$$

Hence, proved.

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Section - C

26.	Let 6 + $7\sqrt{5}$ be rational. Then, there exist coprime <i>a</i> and b ($b \neq 0$), such that			
	$\Rightarrow \qquad 6 + 7\sqrt{5} = \frac{a}{b}, a \text{ and } b \text{ are integers}$			
	$\Rightarrow \qquad 7\sqrt{5} = \frac{a}{b} - 6$			
	$\Rightarrow \qquad 7\sqrt{5} = \frac{a-6b}{b}$			
	$\Rightarrow \qquad \sqrt{5} = \frac{a-6b}{7b}$			
	Since, <i>a</i> and <i>b</i> are integers			
	$\therefore \qquad \qquad \frac{a-6b}{7b} \text{ is an integer}$			
$\Rightarrow \sqrt{5}$ is a rational, which is a contradiction because $\sqrt{5}$ is an irrra				
	Hence, our asumption is wrong. $\therefore 6 + 7\sqrt{5}$ is an irrational number.			
27.	Let $p(y) = 6y^2 - 7y + 2$			
	α and β are zeroes of the above polynomial.			
	Here, $a = 6$, $b = -7$ and $c = 2$.			
	$\Rightarrow \qquad \text{Sum of the roots } = \alpha + \beta = \frac{-b}{a} = \frac{-(-7)}{6} = \frac{7}{6}$			
	Product of zeroes $= \alpha\beta = \frac{c}{a} = \frac{2}{6} = \frac{1}{3}$			
	We are required to find quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.			
	Sum of zeroes $=\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{7 \times 3}{6 \times 1} = \frac{7}{2}$			
	Product of zeroes $=\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{\frac{1}{2}} = 3$			
	Polynomial $p(x) = x^2 - (\text{Sum of zeroes})x + \text{product of zeroes}$			
	$\Rightarrow \qquad p(x) = x^2 - \frac{7x}{2} + 3$			
28.	Let the present ages of two children be x and y .			
	Then according to question:			
	Present age of father $= 2(x + y)$ (1)			
	After 20 years,			
	Ages of the children $= x + 20$ and $y + 20$			
	Age of the father $= 2(x + y) + 20$			
	\Rightarrow According to Question:			
	Age of the father = Sum of the ages of the two children			
	2(x + y) + 20 = (x + 20) + (y + 20)			

	$\Rightarrow \qquad 2x + 2y + 20$	= x + y + 40				
	\Rightarrow $x + y$	= 20				
	Father's age will be $2(x + y)$	= 2(20) = 40 years				
or						
28.	8. Let fixed charge be $\gtrless x$ and charge for distance covered by $\gtrless y/km$.					
	According to the questionn,					
	x + 13y	= 129(1)				
	x + 22y	= 210(2)				
	Subtracting equation (1) from equation (2), we get					
	0	= 81				
		$=\frac{81}{9}=9$				
	Putting $y = 9$ in (1) we get					
	x + 13(9)					
		= 129 - 117 = 12				
	\therefore For travelling 32 km, a pe	erson will have to pay = $\overline{\mathbf{x}}$ (<i>x</i> + 32 <i>y</i>)				
20	0	=₹[12+32(9)] =₹300				
29.	$\sec \theta - \tan \theta$					
	$\Rightarrow \qquad \frac{(\sec\theta - \tan\theta)}{(\sec\theta + \tan\theta)}$	$\times (\sec \theta + \tan \theta) = x$				
	$\Rightarrow \qquad \frac{\sec^2\theta - \tan^2\theta}{\sec\theta + \tan\theta}$	= <i>x</i>				
	$\Rightarrow \qquad \frac{1}{\sec\theta + \tan\theta}$	$= x \qquad [\because \sec^2 \theta - \tan^2 \theta = 1]$				
	\Rightarrow sec θ + tan θ	$=\frac{1}{x} \qquad \dots (2)$				
	Hence, proved.					
	Adding equations (1) and (2)				
	2 sec θ	$=x+rac{1}{x}$				
	\Rightarrow sec θ	$=\frac{x^2+1}{2x}$				
	$\Rightarrow \cos \theta$	$=\frac{2x}{x^2+1}$				
	$\sin^2 \theta$	$= 1 - \cos^2 \theta$				
		$= 1 - \frac{4x^2}{(x^2 + 1)^2}$				
	$=\frac{(x^2+1)^2-4x^2}{(x^2+1)^2}$					
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$$= \frac{x^4 + 1 + 2x^2 - 4x^2}{(x^2 + 1)^2}$$

$$= \frac{x^4 - 2x^2 + 1}{(x^2 + 1)^2}$$

$$= \frac{(x^2 - 1)^2}{(x^2 + 1)^2}$$

$$\Rightarrow \qquad \sin^2 \theta = \frac{(x^2 - 1)^2}{(x^2 + 1)^2}$$

$$\Rightarrow \qquad \sin^2 \theta = \frac{x^2 - 1}{x^2 + 1}$$
30. Let number of black balls be x.

$$\Rightarrow \qquad \text{Number of white balls} = 15$$

$$Total balls = x + 15$$

$$P(getting a black ball) = \frac{15}{x + 15}$$

$$P(getting a white ball) = \frac{15}{x + 15}$$

$$P(getting a white ball) = \frac{15}{x + 15}$$

$$P(getting a white ball) = \frac{15}{x + 15}$$

$$According to the question,$$

$$\frac{x}{x + 15} = 3\left(\frac{15}{x + 15}\right)$$

$$\Rightarrow \qquad x = 45$$

$$\therefore There are 45 black balls.
31. Since the tangent to a circle is perpendicular to the radius through the point of contact.
$$\therefore \qquad \angle OQB = \angle OPB = 90^\circ \qquad ...(1)$$

$$\angle B = 90^\circ \qquad [Given] \dots (2)$$

$$OQ = OP = r \qquad [Radii of a circle] \dots (3)$$

$$\therefore OQBP is a square. \qquad [Using (1), (2) and (3)] \dots (4)$$
Since the tangents from an external point to a circle are equal

$$\therefore \qquad AQ = AR \qquad [Tangents from A] \dots (5)$$
and $DR = DS \qquad [Tangents from A] \dots (5)$
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$$AD - AR = 5 \text{ cm} \qquad \Rightarrow \qquad 23 \text{ cm} - AR = 5 \text{ cm}$$

$$\Rightarrow \qquad AD - AR = 5 \text{ cm} \qquad \Rightarrow \qquad AR = 18 \text{ cm}$$

$$\Rightarrow \qquad AB - BQ = 18 \text{ cm}$$$$

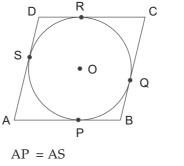
29 cm - BQ = 18 cm \Rightarrow BQ = (29 - 18) cm \Rightarrow BQ = 11 cm... (6) \Rightarrow OQBP is a square [From (4)] Its sides OQ and BQ are equal -[Using (6)]

$$\Rightarrow \qquad \text{Radius OQ} = \text{BQ} = 11 \text{ cm}$$

Hence, the radius (*r*) of the circle is 11 cm.

or

31. Let ABCD be a parallelogram circumscribing a circle with centre O.



$$AP = AS \qquad \dots(1)$$

$$BP = BQ \qquad \dots(2)$$

$$CR = CQ \qquad \dots(3)$$

$$DR = DS \qquad (4)$$

DR = DS...(4)

[Tangents drawn to a circle from an exterior point are equal in length] Adding (1), (2), (3), and (4) we get

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$
$$AB + CD = AD + BC \qquad \dots (5)$$

Since ABCD is a parallelogram

:..

:..

from (5) we get

2 AB = 2 ADAB = ADAB = AD = BC = DC

Since a parallelogram with all sides equal is a rhombus. Thus, ABCD is a rhombus.

Section - D

AB = CD and AD = BC

32. Let length be *x* units and breadth be *y* units.

Area of the rectangle = xy sq units. \Rightarrow

According to the question,

(x+2)(y-2) = xy - 28xy - 2x + 2y - 4 = xy - 28

$$\Rightarrow$$

-2x + 2y + 24 = 0 \Rightarrow 2x - 2y = 24...(1) \Rightarrow Also, (x-1)(y+2) = xy + 33xy + 2x - y - 2 = xy + 33 \Rightarrow 2x - y = 35 \Rightarrow ...(2) Subtracting equation (2) from equation (1), we get -y = 11y = 11 units . . Putting y = 11 in eqn. (2), 2x - 11 = 352x = 46 $x = \frac{46}{2} = 23$ Length of rectangle = 23 units *:*.. breadth of rectangle = 11 units Area of the rectangle $= l \times b$ *:*.. $= 23 \times 11 = 253$ sq units or 32. Let the two numbers be *x* and *y* respectively. According to the question x + y = 1000...(1) $(x^2 - y^2) = 144000$ Also, ...(2) Solving eqn (2), (x - y) (x + y) = 144000 \Rightarrow (1000) (x - y) = 144000[From eqn (1)] \Rightarrow x - y = 144 \Rightarrow ...(3) Adding eqns (1) and (3) we get 2x = 144 + 10002x = 1144 \Rightarrow x = 572.... Putting the value of x = 572 in eqn (1), we get 572 + y = 1000y = 1000 - x \Rightarrow = 1000 - 572= 428Hence, the two numbers are 572 and 428. 33. GIVEN: ΔABC in which DE || BC and DE intersects AB at D and AC at E.

TO PROVE: $\frac{AD}{DB} = \frac{AE}{EC}$

CONSTRUCTION: Join BE, CD and draw $EF \perp AB$, $DN \perp AC$.

PROOF:
$$\frac{\operatorname{ar}(\Delta ADE)}{\operatorname{ar}(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times BD \times EF} = \frac{AD}{BD} \dots (1)$$

 $\frac{AD}{DB} = \frac{AE}{EC}$

§ D

and $\frac{\operatorname{ar}(\Delta ADE)}{\operatorname{ar}(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN} = \frac{AE}{EC} \dots (2)$

But \triangle BDE and \triangle CDE are on the same base DE and between the same parallels DE and BC.

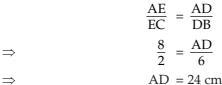
$$\therefore \qquad ar(\Delta BDE) = ar(\Delta CDE) \qquad \dots (3)$$

$$\therefore \qquad \frac{\operatorname{ar}(\Delta ADE)}{\operatorname{ar}(\Delta BDE)} = \frac{\operatorname{ar}(\Delta ADE)}{\operatorname{ar}(\Delta CDE)} \qquad [Using (3)] \qquad \dots (4)$$

Hence,

 \Rightarrow

[From (1), (2) and (4)]



[By Basic Proportionality theorem]

34. Radius of cylindrical tank, $r = \frac{2}{2}$ m = 1 m

Height of cylindrical tank, h = 5 m Volume of cylindrical tank,

$$V = \pi r^2 h = \frac{22}{7} \times (1)^2 \times 5 \text{ m}^3 = \frac{22}{7} \times 5 \text{ m}^3$$

Length of the park, l = 25 m

Breadth, b = 20 m

Let h be height of standing water in the park

 \Rightarrow Volume of cylindrical tank = Volume of water in the park

$$\frac{22}{7} \times 5 = l \times b \times h$$

$$= 25 \times 20 \times h$$
$$h = \frac{22 \times 5}{7 \times 25 \times 20}$$
$$= \frac{11}{350} = 0.031 \text{ m}$$

or

Hence, height of standing water in the park is 0.031 m.

Radius of the tent = $\frac{30}{2}$ = 15 m 34. Height of cylindrical part = 5.5 m Е Height of conical part = 8.25 m - 5.5 m8.25 m⁻ = 2.75 m

Slant height of the conical p

 \Rightarrow

part,

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{(2.75)^2 + (15)^2}$$

$$= \sqrt{\left(\frac{11}{4}\right)^2 + (15)^2} = \sqrt{\frac{121}{16} + 225}$$

$$= \sqrt{\frac{3721}{16}} = \frac{61}{4} \text{ m}$$

D

С

5.5 m -

Total surface area of the tent = CSA of cylindrical part + CSA of conical part

$$= 2\pi rh + \pi rl$$

$$= \pi r + (2h + l)$$

$$= \frac{22}{7} \times 15\left(2 \times \frac{11}{2} + \frac{61}{4}\right)$$

$$= \frac{22}{7} \times 15\left(11 + \frac{61}{4}\right)$$

$$= \frac{22}{7} \times 15\left(\frac{44 + 61}{4}\right)$$

$$= \frac{22}{7} \times 15 \times \frac{105}{4} = \frac{2475}{2} \text{ m}^2$$
Area of canvas used
$$= \frac{2475}{2} \text{ m}^2$$
Length of canvas used
$$= \frac{Area \text{ of canvas}}{\text{Breadth of canvas}}$$

$$= \frac{2475}{2 \times 1.5} = 825 \text{ m}$$

~

35. We prepare the cumulative frequency table, as given below.

Class Interval	Frequency f _i	Cumulative frequency (cf)
0 - 10	5	5
10 – 20	x	5 + x
20 - 30	6	11 + <i>x</i>
30 - 40	y	11 + x + y
40 - 50	6	17 + x + y
50 - 60	5	22 + x + y

Since, the total number of observations is 40,

$$\therefore 22 + x + y = 40$$

$$\Rightarrow x + y = 18 \qquad \dots(1)$$
Median is 31. So median class is 30-40
$$l = 30, cf = 11 + x, f = y \text{ and } h = 10$$

$$Median = l + \frac{\left[\frac{n}{2} - cf\right] \times h}{f}$$

$$\Rightarrow 31 = 30 + \left(\frac{20 - (11 + x)}{y}\right) \times 10$$

$$\Rightarrow 31 - 30 = \frac{20 - 11 - x}{y} \times 10$$

$$\Rightarrow \frac{1}{10} = \frac{9 - x}{y}$$

$$\Rightarrow y = 90 - 10x \qquad \dots(2)$$
Putting the value of y in eqn (1), we get
$$x + 90 - 10x = 18$$

$$-9x = -72$$

$$x = \frac{72}{9} = 8$$

Hence, the values of x and y are 8 and 10 respectively.

Section - E

y = 90 - 10x

= 90 - 80 = 10

36. (a) Total length of silver wire required = Circumference + 4d = $2\pi r + 4d$

...

$$= 2 \times \frac{22}{7} \times 14 \text{ mm} + 4 \times 28 \text{ mm}$$
$$= 88 + 112 = 200 \text{ mm}$$
$$\theta = 45^{\circ}$$

Area of sector
$$= \frac{\theta}{360^{\circ}} \times \pi r^{2}$$
$$= \frac{45}{360} \times \frac{22}{7} \times 14 \times 14 \text{ mm}^{2}$$
$$= 77 \text{ mm}^{2}$$

(*c*) Let the number of revolutions = x

Number of revolutions × circumference = 112π

$$\Rightarrow \qquad x \times 2\pi r = 112\pi$$
$$x = \frac{112\pi}{2\pi r} = \frac{112}{2 \times 14} = 4$$

 \therefore Number of revolutions = 4

or

(c) The circumference of silver part = 44 mm Let radius of silver part = r_1

$$2\pi r_1 = 44 \text{ mm}$$

$$r_1 = \frac{44 \times 7}{2 \times 22} \text{ mm}$$

$$r_1 = 7 \text{ mm}$$

The gold part is 3 mm wide everywhere.

 \therefore Radius of brooch B = 7 mm + 3 mm = 10 mm

Number of revolutions × Circumference = 80π

$$\Rightarrow \qquad x \times 2\pi r = 80\pi$$
$$\Rightarrow \qquad x = \frac{80\pi}{2\pi r} = \frac{80\pi}{2\pi \times 10} = 4$$

 \therefore Number of revolutions = 4

37. (a) The time taken to run 200 m forms the following series.

51, 49, 47, 45, ...

In the above series

$$a_2 - a_1 = a_3 - a_2 = d$$

 \therefore It is an AP.

...

(*b*) The *n*th term of an AP is given by

 $a_n = 2n + 3$ ∴ $a_1 = 2 \times 1 + 3 = 5$ $a_2 = 2 \times 2 + 3 = 7$ Common difference, $d = a_2 - a_1 = 7 - 5 = 2$

(c)

$$a_n = a + (n-1)d$$

$$\Rightarrow \qquad 31 = 51 + (n-1)(-2)$$

$$\Rightarrow \qquad 31 - 51 = -2n + 2$$

$$\Rightarrow \qquad -20 = -2n + 2$$

$$\Rightarrow \qquad -20 - 2 = -2n$$

$$\Rightarrow \qquad -22 = -2n$$

$$\Rightarrow \qquad 11 = n$$

He needs to practice for 11 days.

or

(c)

$$n = 9, a_n = 31, a = 51$$
We need to find d.

$$31 = 51 + (9 - 1)d$$

$$31 - 51 = (9 - 1)d$$

$$\Rightarrow -20 = 8d$$

$$\Rightarrow \frac{-20}{8} = d$$

$$\Rightarrow d = 2.5$$

He should reduce 2.5 s each day.

38. (a) Angle of Depression

(b)
$$\tan 60^\circ = \frac{42}{d}$$
$$\Rightarrow \qquad d = \frac{42}{\sqrt{3}}$$
$$= \frac{42\sqrt{3}}{3}$$
$$= 14\sqrt{3} \text{ m}$$
(c)
$$\tan 60^\circ = \frac{h}{d}$$
$$\Rightarrow \qquad \sqrt{3} = \frac{h}{20}$$
$$h = 20\sqrt{3} \text{ m}$$
(c)
$$\tan \theta = \frac{h}{d}$$

$$\Rightarrow \qquad \tan \theta = \frac{42}{42}$$
$$\Rightarrow \qquad \tan \theta = 1$$



