Sample Question Paper

Basic (Code 041)

ANSWERS

Section - A

1. (*d*)	2. (*a*)	3. (*c*)
4. (*c*)	5. (*b*)	6. (*c*)
7. (*d*)	8. (*a*)	9. (*c*)
10. (*d*)	11. (*a*)	12. (*b*)
13. (*d*)	14. (*d*)	15. (*c*)
16. (*a*)	17. (*b*)	18. (*a*)
19. (*d*)	20. (*b*)	

Section - B

21. Let us assume on the contrary that $\sqrt{3}$ is a rational number and its simplest form is $\frac{a}{b}$, where a and b are integers having no common factor other than 1 and $b \neq 0$.

$\therefore \quad \sqrt{3} = \frac{a}{b}$

$\Rightarrow \quad 3 = \frac{a^2}{b^2}$ [Squaring both sides]

$\Rightarrow \quad 3b^2 = a^2$ ...(1)

$\therefore$ a^2 is divisible by 3 [$\because$ $3b^2$ is divisible by 3]

$\Rightarrow$ a is divisible by 3 [$\because$ 3 is prime and divides $a^2 \Rightarrow$ 3 divides a]

Let $a = 3c$ for some integer c.

Substituting $a = 3c$ in (1), we get

$\Rightarrow \quad 3b^2 = (3c)^2 \Rightarrow 3b^2 = 9c^2 \Rightarrow b^2 = 3c^2$

$\Rightarrow$ b^2 is divisible by 3 [$\because$ $3c^2$ is divisible by 3]

$\Rightarrow$ b is divisible by 3 [$\because$ 3 is prime and divides $b^2 \Rightarrow$ 3 divides b]

Since, a and b are both divisible by 3, $\therefore$ 3 is a common factor of a and b.

But this contradicts the fact a and b have no common factor other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational.

Hence, $\sqrt{3}$ is an irrational number.

22. $$\tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan(A - B) = \tan 30^\circ$$

$$\Rightarrow \quad A - B = 30^\circ \qquad ...(1)$$

$$\sin(A + B) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \sin(A + B) = \sin 60^\circ$$

$$\Rightarrow \quad A + B = 60^\circ \qquad ...(2)$$

Adding (1) and (2), we get

$$\Rightarrow \quad 2A = 90^\circ$$

$$\Rightarrow \quad A = 45^\circ$$

Substituting A = 45° in (1), we get

$$\therefore \quad B = 60^\circ - A \quad \Rightarrow \quad B = 60^\circ - 45^\circ = 15^\circ$$

or

$$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$$

$$\Rightarrow \quad \sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

$$\Rightarrow \quad \sin\theta = (\sqrt{2} - 1)\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{\sin\theta}{\sqrt{2} - 1}$$

$$\Rightarrow \quad \cos\theta = \frac{\sin\theta}{(\sqrt{2} - 1)} \times \frac{(\sqrt{2} + 1)}{(\sqrt{2} + 1)}$$

$$\Rightarrow \quad \cos\theta = \frac{\sin\theta(\sqrt{2} + 1)}{2 - 1}$$

$$\Rightarrow \quad \cos\theta = \sqrt{2}\sin\theta + \sin\theta$$

$$\Rightarrow \quad \cos\theta - \sin\theta = \sqrt{2}\sin\theta$$ **Hence, proved.**

23. Distance covered is 1 revolution of wheel

$$= 2\pi r$$

$$= 2 \times \frac{22}{7} \times \frac{70}{2} = 220 \text{ cm}$$

According to the question, required speed of the bicycle

$$= 19.8 \text{ km/hr} = 5.5 \text{ m/s}$$

Distance covered by the wheel in 10 seconds

$$= \text{Speed} \times \text{Time}$$
$$= 5.5 \times 10 = 55 \text{ m}$$

Distance covered by the wheel in 10 seconds

$$= \text{Number of revolutions} \times \text{Distance covered in 1 revolution}$$

$$\Rightarrow \quad 55 = \text{Number of revolutions} \times \frac{220}{100} \qquad \left[1 \text{ cm} = \frac{1}{100} \text{ m}\right]$$

$$\Rightarrow \quad \text{Number of revolutions} = \frac{55}{220} \times 100 = 25$$

or

Radius of sector, $r = 60$ cm

Angle of major sector, $\theta = 360° - 90° = 270°$

Perimeter of table top $=$ Perimeter of major sector $+ 2r$

$$= \frac{\theta}{360°} \times 2\pi r + 2r$$

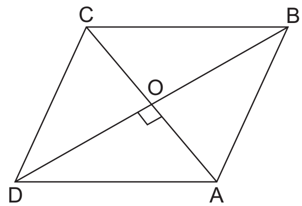
$$= \frac{270°}{360°} \times 2 \times \frac{22}{7} \times 60 + 2 \times 60$$

$$= 282.85 + 120 = 402.85 \text{ cm}$$

24. Let ABCD be the rhombus, whose diagonals AC and BD bisect at O.

Diagonals of rhombus bisect each other at right angle.

Let AC = 50 cm and BD = 120 cm, $\angle AOB = 90°$



So, $AO = \frac{1}{2} AC$ and $OB = \frac{1}{2} BD$

$\Rightarrow$ $AO = \frac{1}{2} \times 50$ and $OB = \frac{1}{2} \times 120$

$\therefore$ AO = 25 cm, OB = 60 cm

In right ΔAOB, we have

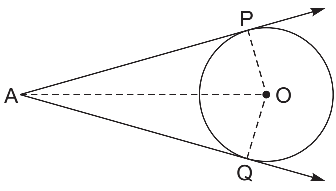
$$AB^2 = AO^2 + OB^2 \quad \Rightarrow \quad AB^2 = 25^2 + 60^2 = 4225$$

$\Rightarrow$ $AB = 65$

Hence, side of the rhombus is 65 cm.

25. **Given:** A circle with centre O and a point A outside it. AP and AQ are tangents drawn to the circle from point A.

To prove: $\angle OAP = \angle OAQ$



Proof: In ΔOAP and ΔOAQ, we have

$$OP = OQ \quad \text{[Radii of the same circle]}$$
$$AP = AQ \quad \text{[Tangents from an external point are equal]}$$
$$OA = OA \quad \text{[Common]}$$
$$\Delta OAP \cong \Delta OAQ \quad \text{[By SSS congruence]}$$
$$\therefore \quad \angle OAP = \angle OAQ \quad \text{[CPCT]}$$

Section - C

26. Given that $x + yp^{1/3} + zp^{2/3} = 0$ ...(1)

On multiplying both sides by $p^{1/3}$, we get

$$xp^{1/3} + yp^{2/3} + zp = 0 \quad ...(2)$$

Multiply (1) by y and (2) by z and subtracting, we get

$\Rightarrow \quad (xy + y^2p^{1/3} + yzp^{2/3}) - (xzp^{1/3} + yzp^{2/3} + z^2p) = 0$

$\Rightarrow \quad (y^2 - xz)p^{1/3} + xy - z^2p = 0$

$\Rightarrow \quad y^2 - xz = 0 \quad \text{and} \quad xy - z^2p = 0$ [$\because p^{1/3}$ is irrational]

$\Rightarrow \quad y^2 = xz \quad \text{and} \quad xy = z^2p$

$\Rightarrow \quad y^2 = xz \quad \text{and} \quad x^2y^2 = z^4p^2$

$\Rightarrow \quad x^2(xz) = z^4p^2$

$\Rightarrow \quad x^3z - z^4p^2 = 0 \quad \Rightarrow \quad z(x^3 - z^3p^2) = 0$

$\Rightarrow \quad x^3 - z^3p^2 = 0 \quad \text{or} \quad z = 0$

$\Rightarrow \quad p^2 = \dfrac{x^3}{z^3}$

$\Rightarrow \quad p^{2/3} = \dfrac{x}{z}$

This is not possible as $p^{1/3}$ is irrational and $\dfrac{x}{z}$ is rational.

$\therefore \quad x^3 - p^2z^3 \neq 0$

Hence, $\quad z = 0$

Putting $z = 0$ in $y^2 - xz = 0$, we get $y = 0$

Putting $y = 0$ and $z = 0$ in (1)

$\Rightarrow \quad x = 0$

Hence, $\quad x = y = z = 0.$

27. Sum of roots (S) = –1

Product of roots (P) = –6

Required quadratic polynomial

$$x^2 - Sx + P$$
$$= x^2 - (-1)x + (-6)$$
$$= x^2 + x - 6$$

28. Let 1 man can finish the work in x days

and 1 woman can finish it in y days.

1 man's 1 day's work = $\frac{1}{x}$

1 woman's 1 day's work = $\frac{1}{y}$

8 men and 12 women can finish the work in 10 days

$\Rightarrow$ (8 men's 1 day's work) + (12 women's 1 day's work) = $\frac{1}{10}$

$$\Rightarrow \quad \frac{8}{x} + \frac{12}{y} = \frac{1}{10} \quad ...(1)$$

6 men and 8 women can finish the work in 14 days

$\Rightarrow$ (6 men's 1 day's work) + (8 women's 1 day's work) = $\frac{1}{14}$

$$\Rightarrow \quad \frac{6}{x} + \frac{8}{y} = \frac{1}{14} \quad ...(2)$$

Multiply equation (1) by 2 and equation (2) by 3

$$\Rightarrow \quad \frac{16}{x} + \frac{24}{y} = \frac{1}{5} \quad ...(3)$$

$$\Rightarrow \quad \frac{18}{x} + \frac{24}{y} = \frac{3}{14} \quad ...(4)$$

Subtracting (3) from (4)

$$\Rightarrow \quad \frac{18}{x} - \frac{16}{x} = \frac{3}{14} - \frac{1}{5}$$

$$\Rightarrow \quad \frac{2}{x} = \frac{1}{70} \quad \Rightarrow \quad x = 140$$

From equation (2)

$$\frac{6}{140} + \frac{8}{y} = \frac{1}{14}$$

$$\Rightarrow \quad \frac{8}{y} = \frac{1}{14} - \frac{6}{140}$$

$$\Rightarrow \quad \frac{8}{y} = \frac{4}{140}$$

$$\Rightarrow \quad y = 280$$

Hence, time taken by man alone to finish the work is 140 days, and time taken by woman alone to finish the work is 280 days.

or

$$\frac{x}{3} + \frac{y}{4} = 6 \quad ...(1)$$

$$\frac{x}{6} + \frac{y}{2} = 6 \quad ...(2)$$

Dividing equation (1) by 2, we get

$$\Rightarrow \qquad \frac{x}{6}+\frac{y}{8} = 3 \qquad ...(3)$$

Subtracting equation (3) from equation (2), we get

$$\frac{x}{6}+\frac{y}{2}-\frac{x}{6}-\frac{y}{8} = 6-3$$

$$\Rightarrow \qquad \frac{y}{2}-\frac{y}{8} = 3$$

$$\Rightarrow \qquad \frac{3y}{8} = 3$$

$$\Rightarrow \qquad y = 8$$

Substituting $y = 8$ in equation (1), we get

$$\frac{x}{3}+\frac{8}{4} = 6$$

$$\Rightarrow \qquad \frac{x}{3}+2 = 6$$

$$\Rightarrow \qquad \frac{x}{3} = 4 \quad \Rightarrow \quad x = 12$$

$$\therefore \qquad 3y-2x = 3\times 8-2\times 12 = 0$$

and $\quad \dfrac{x}{y}+\dfrac{1}{2}$

$$= \frac{12}{8}+\frac{1}{2}$$

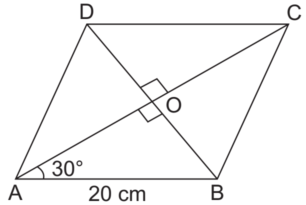
$$= \frac{3}{2}+\frac{1}{2}$$

$$= 2$$

29. In the given diagonals AC and BD bisect each other at right angles at O.

$$\Delta AOB \cong \Delta AOD \qquad \text{[By RHS congruence]}$$

$$\therefore \qquad \angle BAO = \angle DAO = \frac{1}{2}\angle DAB = \frac{1}{2}\times 60^\circ = 30^\circ \qquad \text{[CPCT]}$$



In right ΔAOB,

$$\sin 30^\circ = \frac{OB}{AB} \quad \Rightarrow \quad \frac{1}{2}=\frac{OB}{20} \quad \Rightarrow \quad OB = 10 \text{ cm}$$

$$BD = 2OB \quad \Rightarrow \quad BD = 2\times 10 = 20 \text{ cm}$$

$$\cos 30^\circ = \frac{AO}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AO}{20} \Rightarrow AO = 10\sqrt{3} \text{ cm}$$

$$AC = 2AO \Rightarrow AC = 2 \times 10\sqrt{3} = 20\sqrt{3} \text{ cm}$$

Hence, length of diagonals are 20 cm and $20\sqrt{3}$ cm.

30. Class size = Difference between any two consecutive mid values = 25 – 15 = 10.

Mid values 15 corresponds to class (15 – 5) – (15 + 5), *i.e.* 10 – 20, and so on.

Thus, cumulative frequency table for the given data is:

Class Interval	***Frequency*** (f_i)	***Cumulative Frequency*** (cf)
10 – 20	4	4
20 – 30	28	32
30 – 40	15	47
40 – 50	20	67
50 – 60	17	84
60 – 70	16	100
Total	$n = \Sigma f_i = 100$	

Here, $n = \Sigma f_i = 100 \Rightarrow \frac{n}{2} = \frac{100}{2} = 50.$

The cumulative frequency just greater than 50 is 67 and the corresponding class is 40 – 50. So, the median class is 40 – 50.

$\therefore$ $l = 40, cf = 47, f = 20$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 40 + \left(\frac{50 - 47}{20}\right) \times 10 = 40 + \frac{3}{20} \times 10$$

$$= 40 + 1.5$$

$$= 41.5$$

Hence, the median is 41.5.

31. $\angle PQO = 90^\circ$ ...(1) [The tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\angle POQ + \angle POR = 180^\circ$ [Straight angle]

$\Rightarrow$ $\angle POQ + 130^\circ = 180^\circ$

$\Rightarrow \quad \angle POQ = 50°$

In ΔOPQ, we have

$\angle OPQ + \angle PQO + \angle POQ = 180°$ [Sum of angles of a triangle is 180°]

$\Rightarrow \quad \angle 1 + 90° + 50° = 180°$ [Using (1) and (2)]

$\Rightarrow \quad \angle 1 = 40°$ ...(3)

$\angle RST = \frac{1}{2} \angle TOR$

[Angle subtended by an arc of a circle at the center is twice the angle subtended by it at any point on remaining part of the circle]

$\Rightarrow \quad \angle 2 = \frac{1}{2} \angle POR$

$\Rightarrow \quad \angle 2 = \frac{1}{2} \times 130° = 65°$ ...(4)

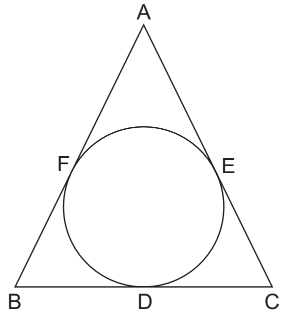
Adding (3) and (4), we get

$\therefore \quad \angle 1 + \angle 2 = 40° + 65° = 105°$

Hence, $\angle 1 + \angle 2 = 105°$

or

Since, tangents drawn from an external point to a circle are equal.



$BF = BD, CD = CE$ and $AE = AF$ ...(1)

$\text{Semi-perimeter} = \frac{AB + BC + CA}{2} = s$

$\Rightarrow \quad AB + BC + CA = 2s$

$\Rightarrow \quad (AF + BF) + (BD + CD) + (CE + AE) = 2s$

$\Rightarrow \quad (AE + BD) + (BD + CE) + (CE + AE) = 2s$ [$\because$ BF = BD, CD = CE, AE = AF]

$\Rightarrow \quad 2(AE + CE) + 2BD = 2s$

$\Rightarrow \quad 2AC + 2BD = 2s$

$\Rightarrow \quad AC + BD = s$

$\Rightarrow \quad BD = s - AC$ **Hence, proved.**

Section - D

32. Let total number of birds be x.

Then, number of birds moving about in lotus plant = $\frac{x}{4}$.

The number of birds moving on a hill = $\frac{x}{9}+\frac{x}{4}+7\sqrt{x}$

Number of birds remain on trees = 56

$\therefore$ Number of birds moving in lotus plant + Birds moving on hill + Birds remain on trees = Total number of birds

$$\Rightarrow \quad \frac{x}{4}+\frac{x}{9}+\frac{x}{4}+7\sqrt{x}+56 = x$$

$$\Rightarrow \quad \left(\frac{x}{4}+\frac{x}{9}+\frac{x}{4}-x\right)+7\sqrt{x}+56 = 0$$

$$\Rightarrow \quad \left(\frac{9x+4x+9x-36x}{36}\right)+7\sqrt{x}+56 = 0$$

$$\Rightarrow \quad \frac{-14x}{36}+7\sqrt{x}+56 = 0$$

$$\Rightarrow \quad x-18\sqrt{x}-144 = 0 \qquad \left[\text{Multiply by } \frac{-36}{14}\right]$$

Let $\sqrt{x} = y$

$$\Rightarrow \quad y^2 - 18y - 144 = 0$$

$$\Rightarrow \quad y^2 - 24y + 6y - 144 = 0 \quad \Rightarrow \quad y(y-24)+6(y-24) = 0$$

$$\Rightarrow \quad (y+6)(y-24) = 0$$

$$\Rightarrow \quad y = -6 \quad \text{or} \quad y = 24$$

$$\therefore \quad \sqrt{x} = -6 \quad \text{or} \quad \sqrt{x} = 24$$

Since, $\sqrt{x} = -6$ is not possible

$$\therefore \quad x = (24)^2 = 576$$

Hence, the total number of birds is 576, which is in accordance with problem.

or

The given equation is

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a) = 0$$

$$\Rightarrow \quad 3x^2 - 2(a+b+c)x + (ab+bc+ca) = 0$$

Comparing the given equation with $a_1x^2 + b_1x + c_1 = 0$, we have

$$a_1 = 3, \; b_1 = -2(a+b+c), \; c_1 = ab+bc+ca$$

$$\begin{aligned} D &= b_1^2 - 4a_1c_1 \\ &= 4(a+b+c)^2 - 4 \times 3(ab+bc+ca) \\ &= 4(a^2+b^2+c^2+2ab+2bc+2ca) - 12(ab+bc+ca) \end{aligned}$$

$$= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 3ab - 3bc - 3ca)$$
$$= 4(a^2 + b^2 + c^2 - ab - bc - ca)$$
$$= 2(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$$
$$= 2[(a^2 + b^2 - 2ab) - (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ca)]$$
$$= 2[(a - b^2) + (b - c)^2 + (c - a)^2]$$
$$[\because \quad (a - b)^2 \geq 0, (b - c)^2 \geq 0, (c - a)^2 \geq 0]$$

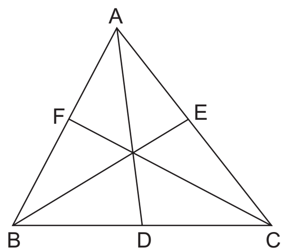
As, $D \geq 0$, roots of the given equation are real.

For equal roots

$$D = 0$$
$$\Rightarrow \quad (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$$
$$\Rightarrow \quad a - b = 0, b - c = 0, c - a = 0 \quad \Rightarrow \quad a = b = c$$
$$\Rightarrow \quad a = b, b = c, c = a$$

Hence, the roots are equal only when $a = b = c$.

33. **Given:** In ΔABC, AD, BE and CF are medians.



To prove: $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$

Proof: Since, in any triangle, sum of squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median bisecting it.

Therefore, we have,

$$AB^2 + AC^2 = 2(AD^2 + BD^2) \quad ...(1)$$
$$BC^2 + AB^2 = 2(BE^2 + CE^2) \quad ...(2)$$
and
$$CA^2 + BC^2 = 2(CF^2 + AF^2) \quad ...(3)$$

Adding the corresponding sides of equations (1), (2) and (3), we get

$$2(AB^2 + BC^2 + CA^2) = 2(AD^2 + BD^2 + BE^2 + CE^2 + CF^2 + AF^2)$$
$$\Rightarrow \quad AB^2 + BC^2 + AC^2 = AD^2 + BE^2 + CF^2 + \left(\frac{BC}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{AB}{2}\right)^2$$

$[\because$ AD, BE and CF are medians $\therefore$ $BD = \frac{1}{2}BC$, $CE = \frac{1}{2}AC$ and $AF = \frac{1}{2}AB$]

$$\Rightarrow \quad AB^2 - \frac{AB^2}{4} + BC^2 - \frac{BC^2}{4} + CA^2 - \frac{CA^2}{4} = AD^2 + BE^2 + CF^2$$
$$\Rightarrow \quad \frac{3}{4}AB^2 + \frac{3}{4}BC^2 + \frac{3}{4}CA^2 = AD^2 + BE^2 + CF^2$$
$$\Rightarrow \quad 3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$$

34. Let R be the radius and H the height of the cylindrical tank.

Then radius (R) of tank $= \frac{10}{2}$ m = 5 m and H = 2 m

$\therefore$ Volume of cylindrical tank = $\pi R^2 H$

$$= \pi \times (5)^2 \times 2 \text{ m}^3 \quad ...(1)$$

Rate of flow of water through the pipe

$$= 6 \text{ km/h} = \frac{6 \times 1000}{60} = 100 \text{ m/min.} \quad ...(2)$$

Let r be the radius of the cross-section of the pipe

$$= \frac{20}{2} = 10 \text{ cm} = \frac{1}{10} \text{ m}$$

Area of cross-section of the pipe = $\pi r^2 = \pi\left(\frac{1}{10}\right)^2 \text{ m}^2$

Volume of water flowing through pipe in 1 minute

$$= \pi\left(\frac{1}{10}\right)^2 \times 100 \text{ m}^2 \qquad \text{[using (2)]} \quad ...(3)$$

Suppose the tank gets filled in x minutes.

$$\begin{bmatrix}\text{Volume of water flowing through} \\ \text{pipe in } x \text{ minutes}\end{bmatrix} = \begin{bmatrix}\text{Volume of water} \\ \text{in fill tank}\end{bmatrix}$$

$$\Rightarrow \quad \pi\left(\frac{1}{10}\right)\left(\frac{1}{10}\right)(100) \times x = \pi(5)^2\,(2)$$

$$\Rightarrow \quad \pi \times \left(\frac{1}{10}\right)^2 (100) \times x = \pi \times (5)^2 \times 2 \qquad \text{[Using (1) and (3)]}$$

$$\Rightarrow \quad x = 5 \times 5 \times 2 = 50$$

Hence, tank will get filled up in 50 minutes.

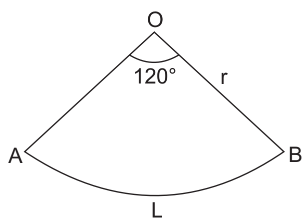
or

Let AOB represent the sector of the circle with radius r = 15 cm and central angle AOB = 120° (= θ say).

Then, length of arc AB = $\frac{\theta}{360} \times 2\pi r$

$$= \left(\frac{120}{360} \times 2 \times \frac{22}{7} \times 15\right) \text{cm} = \frac{220}{7} \text{ cm} \quad ...(1)$$

When sector is rolled up, arc AB forms the circumference of base of the cone and radius (r = 15 cm) forms the slant height of cone.



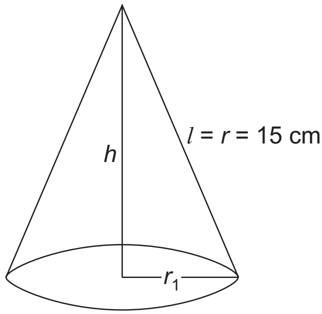
$\therefore$ l = 15 cm

Let r_1 be the radius of cone and h its height.

$$\therefore \quad 2\pi r_1 = \frac{220}{7} \text{ cm} \quad \text{[Using (1)]}$$

$\Rightarrow$ $$r_1 = \frac{220}{7} \times \frac{1}{2} \times \frac{7}{22} = 5 \text{ cm}$$

$$h = \sqrt{l^2 - r_1^2} = \sqrt{15^2 - 5^2} = \sqrt{200} = 10\sqrt{2} \text{ cm}$$



$\therefore$ Volume of cone $= \frac{1}{3}\pi r_1^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 10\sqrt{2} = 370.38 \text{ cm}^3.$$

Hence, the volume of the cone is 370.38 cm^3 (approx.).

35.

Height	*Mid value* (x_i)	*Number of girls* (f_i)	*Cumulative frequency* (*cf*)	$f_i x_i$
120 – 130	125	2	2	250
130 – 140	135	8	10	1080
140 – 150	145	12	22	1740
150 – 160	155	20	42	3100
160 – 170	165	8	50	1320
Total	$n = \Sigma f_i = 50$			$\Sigma f_i x_i = 7490$

Mean: $\frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{7490}{50} = 149.8$

Median: $n = \Sigma f_i = 50$, so $\frac{n}{2} = \frac{50}{2} = 25$

Cumulative frequency just greater than 25 is 42 and the corresponding class is 150 – 160. So the median class is 150 – 160.

$\therefore$ $l = 150$, $cf = 22$, $f = 20$, $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 150 + \left(\frac{25 - 22}{20}\right) \times 10 = 150 + 1.5$$

$$= 151.5$$

Mode: Here the maximum frequency is 20 and class corresponding to it is 150 – 160. So modal class is 150 – 160.

$\therefore$ $l = 150, f_1 = 20, f_0 = 12, f_2 = 8, h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 150 + \left(\frac{20 - 12}{2 \times 20 - 12 - 8}\right) \times 10$$

$$= 150 + \frac{8}{20} \times 10$$

$$= 150 + 4 = 154$$

Hence, mean height of the girls is 149.8 cm, their median height is 151.5 cm and their modal height is 154 cm.

Section - E

36. Let the sale of smartphones in 1st year be a_1 and 2nd year be a_2.

Difference in sale of phones in 2nd year and 1st year is d.

Sale of smartphones in 3rd year, $a_3 = 10{,}000$

Sale of smartphones in 5th year, $a_5 = 14{,}000$

(*a*) Sale in 2nd year = a_2.

$\therefore$ $a_3 = a_1 + (3 - 1)d$ $\quad [a_n = a_1 + (n - 1)d]$

$\Rightarrow$ $10000 = a_1 + 2d$ ...(1)

$a_5 = a_1 + 4d \Rightarrow 14000 = a_1 + 4d$ ...(2)

Subtracting equation (1) from equation (2), we get

$\Rightarrow$ $14000 - 10000 = a_1 + 4d - a_1 - 2d$

$\Rightarrow$ $4000 = 2d \Rightarrow d = \frac{4000}{2} = 2000$

$\therefore$ $10000 = a_1 + 2 \times 2000$

$\Rightarrow$ $a_1 = 10000 - 4000 = 6000$

$a_2 = a_1 + d = 6000 + 2000 = 8000$

Hence, the sale in 2nd year is 8000.

(*b*) Total sale in first 5 years = S_5

$$S_5 = \frac{5}{2}(6000 + 14000) \qquad \left[S_n = \frac{n}{2}(a_1 + a_n)\right]$$

$$= \frac{5}{2} \times 20000 = 50000$$

Hence, total sale of smartphones in first 5 years is 50,000.

or

Let the sale be a_n in n^{th} year.

$\therefore \quad a_n = a_1 + (n-1)d$

$\Rightarrow \quad 22000 = 6000 + (n-1) \times 2000$

$\Rightarrow \quad 22000 - 6000 = (n-1) \times 2000$

$\Rightarrow \quad 16000 = (n-1) \times 2000$

$\Rightarrow \quad 8 = n - 1 \quad \Rightarrow \quad n = 9$

Hence, sale would be 22000 after 9^{th} year.

(*c*) Let the total sale will be S_n in n^{th} years.

$\therefore \quad S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$\Rightarrow \quad 66000 = \frac{n}{2}[2 \times 6000 + (n-1)\,2000]$

$\Rightarrow \quad 66000 = 6000n + (n^2 - n)1000$

$\Rightarrow \quad 66 = 6n + n^2 - n$

$\Rightarrow \quad n^2 + 5n - 66 = 0$

$\Rightarrow \quad n^2 + 11n - 6n - 66 = 0 \quad \Rightarrow \quad (n-6)(n+11) = 0$

$\Rightarrow \quad n = 6, -11$

As –11 can't be possible year.

Hence, total sale would be 66,000 after 6 years.

37. (*a*) Coordinates of C is (–5, –3), A is (2, 7), B is (7, –8)

By distance formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance between A and C $= \sqrt{(2+5)^2 + (7+3)^2}$

$= \sqrt{49 + 100} = \sqrt{149}$ unit

Distance between B and C $= \sqrt{(7+5)^2 + (-8+3)^2}$

$= \sqrt{144 + 25} = \sqrt{169} = 13$ unit

Hence, Dev travelled more distance.

(*b*) By mid-point formula coordinates of mid-point of BA is (x, y)

$$x = \frac{7+2}{2}, y = \frac{-8+7}{2} \qquad \left[x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}\right]$$

$$x = \frac{9}{2}, y = -\frac{1}{2}$$

Hence, coordinates of that place is $\left(\frac{9}{2}, -\frac{1}{2}\right)$

or

Area of triangle formed by points A, B and C is ABC

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} |2(-8-7) + 7(-3-7) + (-5)(7+8)|$$

$$= \frac{1}{2} |2(-15) + 7(-10) - 5 \times 15|$$

$$= \frac{1}{2}(-30 - 70 - 75)$$

$$= \frac{1}{2}(-175) = 87.5 \text{ sq units.}$$

Hence, area of the triangle formed by A, B and C is 87.5 unit square.

(*c*) Distance between A and B = $\sqrt{(7-2)^2 + (-8-7)^2}$

$$= \sqrt{25 + 225}$$

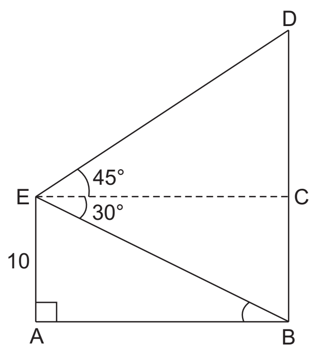
$$= \sqrt{250} \text{ unit} = 5\sqrt{10} \text{ unit}$$

38. Let AE be height of the tree and BD is height of the tower.

AB is the distance between the tree and the tower.

Angle of elevation, $\angle DEC = 45°$

Angle of depression, $\angle CEB = 30°$



(*i*) $\angle ABE = \angle CEB = 30°$ [Alternate angles]

In ΔAEB,

$$\tan 30° = \frac{AE}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{AB} \quad \Rightarrow \quad AB = 10\sqrt{3} \text{ m}$$

Therefore, distance between the tree and the tower = $10\sqrt{3}$ m

(*ii*) In ΔCDE,

$$\tan 45° = \frac{CD}{EC}$$

$\Rightarrow \quad 1 = \dfrac{CD}{10\sqrt{3}}$ [EC = AB]

$\Rightarrow \quad CD = 10\sqrt{3}$ m

$\therefore \quad BD = BC + CD = 10 + 10\sqrt{3} = 10\,(1+\sqrt{3})$ m

Therefore, height of the tower = $10\,(1+\sqrt{3})$ m

Area of trapezium ABDE = $\dfrac{1}{2}$ × [sum of lengths of parallel sides
× distance between them]

$$= \frac{1}{2}\left[10 + 10(1+\sqrt{3})\right] \times 10\sqrt{3} \text{ m}^2$$

$$= \left[20 + 10\sqrt{3}\right] \times 5\sqrt{3} = (100\sqrt{3} + 150) \text{ m}^2.$$

or

$$\sin 45^\circ = \frac{CD}{ED}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{10\sqrt{3} \text{ m}}{ED}$$

$$\Rightarrow \quad ED = 10\sqrt{6} \text{ m}$$

Perimeter of the trapezium = AB + BD + ED + AE

$$= (10\sqrt{3} + 10 + 10\sqrt{3} + 10\sqrt{6} + 10) \text{ m}$$

$$= (20 + 20\sqrt{3} + 10\sqrt{6}) \text{ m}$$

$\therefore$ Perimeter of the trapezium = $10(2 + 2\sqrt{3} + \sqrt{6})$ m.

Hence, perimeter of the trapezium is $10(2 + 2\sqrt{3} + \sqrt{6})$ m.

(*iii*) Height of the tower = $BC + CD = 10 + 10\sqrt{3}$

$$= 10(1+\sqrt{3}) \text{ m.}$$

Hence, height of the tower = $10(1+\sqrt{3})$ m.