Sample Question Paper

Standard (Code 041)

ANSWERS

Section - A

1. (*b*) 2. (*c*) 3. (*c*)
4. (*b*) 5. (*a*) 6. (*a*)
7. (*b*) 8. (*d*) 9. (*c*)
10. (*a*) 11. (*d*) 12. (*c*)
13. (*b*) 14. (*a*) 15. (*d*)
16. (*b*) 17. (*a*) 18. (*c*)
19. (*d*) 20. (*b*)

Section - B

21. Let us assume on the contrary that $\sqrt{7}-\sqrt{2}$ is a rational number. Then, there exist coprime a and b ($b \neq 0$), such that

$$\sqrt{7}-\sqrt{2} = \frac{a}{b} \quad \Rightarrow \quad \frac{a}{b}+\sqrt{2}=\sqrt{7}$$

$$\Rightarrow \quad \left(\frac{a}{b}+\sqrt{2}\right)^2 = (\sqrt{17})^2 \quad \Rightarrow \quad \frac{a^2}{b^2}+\frac{2a\sqrt{2}}{b}+2 = 7$$

$$\Rightarrow \quad \frac{a^2}{b^2}-5 = \frac{-2a\sqrt{2}}{b} \quad \Rightarrow \quad \frac{a^2-5b^2}{b^2} = \frac{-2a\sqrt{2}}{b}$$

$$\Rightarrow \quad \frac{a^2-5b^2}{2ab} = -\sqrt{2} \quad \Rightarrow \quad \sqrt{2} \text{ is a rational number.}$$

This contradicts the fact that $\sqrt{2}$ is irrational.

So, our assumption is wrong.

Hence, $\sqrt{7}-\sqrt{2}$ is irrational.

22. In ΔPST and ΔRQT,

$$\angle PTS = \angle QTR \quad \text{[Vertically opp. } \angle s]$$

$$\angle PST = \angle RQT \quad \text{[PS || QR, alternate angles]}$$

$$\therefore \quad \Delta PST \sim \Delta RQT \quad \text{[By AA criterion of similarity]}$$

$$\Rightarrow \quad \frac{PS}{ST} = \frac{RQ}{TQ} \quad \Rightarrow \quad \frac{18}{13} = \frac{36}{TQ} \quad \Rightarrow \quad TQ = 26 \text{ cm}$$

In ΔQUT and ΔQPS,

$\angle QUT = \angle QPS$ [Corresponding angles, TU || PS]

$\angle QTU = \angle QSP$ [Corresponding angles, TU || PS]

$\therefore$ $\Delta QUT \sim \Delta QPS$ [By AA criterion of similarity]

$\therefore$ $\frac{QT}{QS} = \frac{UT}{PS} \Rightarrow \frac{26}{39} = \frac{UT}{18}$ [QS = ST + QT]

$\Rightarrow$ $UT = 12$ cm

Hence, TQ = 26 cm and UT = 12 cm.

23. Since the tangent at any point of a circle is perpendicular to the radius through the point of contact, therefore $CQ \perp XZ$ and $CP \perp XY$

$\Rightarrow$ $\angle X = \angle CPX = \angle CQX = 90°$

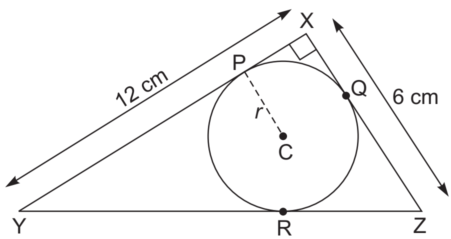
$\therefore$ $CP = XQ = XP = r$ [Radius of circle]

Now, $ZQ = ZR = XZ - XQ = (16 \text{ cm} - r)$ ...(1)

$YR = YP = XY - XP = (12 \text{ cm} - r)$ ...(2)

$\therefore$ $YZ = ZR + YR = 16 \text{ cm} - r + 12 \text{ cm} - r = 28 \text{ cm} - 2r$

[Using (1) and (2)] ...(3)



Now, $YZ^2 = XZ^2 + XY^2$

$\Rightarrow$ $(28 \text{ cm} - 2r)^2 = 16^2 \text{ cm}^2 + 12^2 \text{ cm}^2$ [Using (3)]

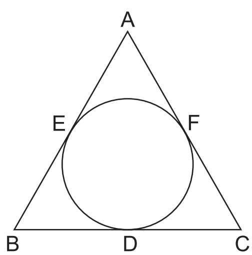
$\Rightarrow$ $(28 \text{ cm} - 2r)^2 = 20^2 \text{ cm}^2$

$\Rightarrow$ $28 \text{ cm} - 2r = 20 \text{ cm} \Rightarrow r = 4 \text{ cm}$

Hence, $r = 4$ cm.

or

Let side BC of isosceles ΔABC touch circle at D.



Given: $AB = AC$

To prove: BD = DC

Proof: Since the lengths of tangents drawn from an external point to circle are equal,

$\therefore$ AE = AF

Similarly, BE = BD and CF = CD

Now, AB – AE = AC – AF

$\Rightarrow$ BE = CF

$\therefore$ BD = DC [$\because$ BE = BD and CF = DC]

Hence, BC is bisected at point of contact.

24. $\text{cosec}\,\theta - \sin\theta = p$ and $\sec\theta - \cos\theta = q$ [Given]

$$\Rightarrow \quad \frac{1}{\sin\theta} - \sin\theta = p \quad \text{and} \quad \frac{1}{\cos\theta} - \cos\theta = q$$

$$\Rightarrow \quad \frac{1-\sin^2\theta}{\sin\theta} = p \quad \text{and} \quad \frac{1-\cos^2\theta}{\cos\theta} = q$$

$$\Rightarrow \quad \frac{\cos^2\theta}{\sin\theta} = p \quad \text{and} \quad \frac{\sin^2\theta}{\cos\theta} = q$$

$$\text{LHS} = p^{4/3}q^{2/3} + p^{2/3}q^{4/3}$$

$$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^{4/3}\left(\frac{\sin^2\theta}{\cos\theta}\right)^{2/3} + \left(\frac{\cos^2\theta}{\sin\theta}\right)^{2/3}\left(\frac{\sin^2\theta}{\cos\theta}\right)^{4/3}$$

$$= \frac{(\cos\theta)^{8/3-2/3}}{(\sin\theta)^{4/3-4/3}} + \frac{(\cos\theta)^{4/3-4/3}}{(\sin\theta)^{2/3-8/3}}$$

$$= \frac{(\cos\theta)^{6/3}}{1} + \frac{1}{(\sin\theta)^{-6/3}}$$

$$= \cos^2\theta + \sin^2\theta = 1 = \text{RHS}$$

Hence, $p^{4/3}q^{2/3} + p^{2/3}q^{4/3} = 1$

or

$$\text{LHS} = \sin^6\theta + \cos^6\theta$$

$$= (\sin^2\theta)^3 + (\cos^2\theta)^3$$

$$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$$

$$= \sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta \qquad [\sin^2\theta + \cos^2\theta = 1]$$

$$= \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta$$

$$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$$

$$= 1^2 - 3\sin^2\theta\cos^2\theta$$

$$= 1 - 3\sin^2\theta\cos^2\theta = \text{RHS}$$

25. Let the diameters of concentric circle be $4d$ and $5d$.

$\therefore$ Radii are $2d$ and $\frac{5}{2}d$.

Ratio of areas of the two regions

$$= \text{Area of region I} : \text{Area of region II}$$
$$= \pi(2d)^2 : \pi\left(\frac{5}{2}d\right)^2 - \pi(2d)^2$$
$$= 4d^2 : \frac{25}{4}d^2 - 4d^2$$
$$= 4d^2 : \frac{25-16}{4}d^2$$
$$= 4d^2 : \frac{9}{4}d^2$$
$$= 16 : 9$$

Hence, the ratio of the areas of these regions is 16 : 9.

Section - C

26. Time required for the bells to ring together is the LCM of 2, 4, 6, 8, 10 and 12 (in minutes).

LCM of 2, 4, 6, 8, 10, 12 = 120

$\therefore$ After every 120 minutes or 2 hours bells toll together

Required number of times = $\frac{12}{2} + 1$ = 7 times

Hence, they rang together between 9 a.m. and 9 p.m. 7 times.

27. Since α and β are the zeroes of the polynomial

$$f(x) = px^2 + qx + r$$
$$\alpha + \beta = \frac{-q}{p}, \alpha\beta = \frac{r}{p} \quad ...(1)$$

$\therefore$ $$(\alpha+\beta)^2 = \frac{q^2}{p^2} \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = \frac{q^2}{p^2}$$

$\Rightarrow$ $$\alpha^2 + \beta^2 + \frac{2r}{p} = \frac{q^2}{p^2}$$

$\Rightarrow$ $$\alpha^2 + \beta^2 = \frac{q^2}{p^2} - \frac{2r}{p} \quad ...(2)$$

Now, $p\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + q\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = p\left(\frac{\alpha^3 + \beta^3}{\alpha\beta}\right) + q\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)$

$$= p\,\frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{\alpha\beta} + q\,\frac{(\alpha^2+\beta^2)}{\alpha\beta}$$

$$= \frac{p\left(\frac{-q}{p}\right)\left(\frac{q^2}{p^2}-\frac{2r}{p}-\frac{r}{p}\right)}{r/p} + \frac{q\left(\frac{q^2}{p^2}-\frac{2r}{p}\right)}{r/p}$$

[Using (1) and (2)]

$$= \frac{(-q)(q^2-3rp)\times p}{p^2\times r} + \frac{q(q^2-2pr)\times p}{p^2\times r}$$

$$= \frac{-q^3+3prq+q^3-2prq}{pr}$$

$$= \frac{prq}{pr} = q$$

28. Let the total number of students be x and number of rows be y.

Number of students in each row = $\frac{x}{y}$

Total number of students = No. of rows × No. of students in each row

$$\Rightarrow \quad x = (y-1)\left(\frac{x}{y}+2\right)$$

$$\Rightarrow \quad x = x + 2y - \frac{x}{y} - 2 \quad \Rightarrow \quad 2y - \frac{x}{y} - 2 = 0 \qquad ...(1)$$

Now, 2 students are removed.

$$x = (y+2)\left(\frac{x}{y}-2\right)$$

$$\Rightarrow \quad x = x - 2y + \frac{2x}{y} - 4 \quad \Rightarrow \quad 2y - \frac{2x}{y} + 4 = 0 \qquad ...(2)$$

Multiplying equation (1) by 2 and then subtracting the result from equation (2), we get,

$$\Rightarrow \quad 2y - \frac{2x}{y} + 4 - \left(4y - \frac{2x}{y} - 4\right) = 0$$

$$\Rightarrow \quad -2y + 8 = 0 \quad \Rightarrow \quad y = 4$$

Substituting $y = 4$ in equation (1), we get

$$2\times 4 - \frac{x}{4} - 2 = 0$$

$$\Rightarrow \quad 6 - \frac{x}{4} = 0 \quad \Rightarrow \quad x = 24$$

Hence, the total number of students in parade is 24.

or

Suppose 1 man can finish the work in x days and 1 woman can finish it in y days.

$$1 \text{ man's } 1 \text{ day work} = \frac{1}{x}$$

$$1 \text{ woman's } 1 \text{ day work} = \frac{1}{y}$$

8 men and 12 women can finish the work in 10 days

$$\therefore \quad \frac{8}{x} + \frac{12}{y} = \frac{1}{10} \quad \text{...(1)}$$

6 men and 8 women can finish the work in 14 days

$$\therefore \quad \frac{6}{x} + \frac{8}{y} = \frac{1}{14} \quad \text{...(2)}$$

Multiply equation (1) by 2 and equation (2) by 3

$$\Rightarrow \quad \frac{16}{x} + \frac{24}{y} = \frac{1}{5} \quad \text{...(3)}$$

$$\Rightarrow \quad \frac{18}{x} + \frac{24}{y} = \frac{3}{14} \quad \text{...(4)}$$

Subtracting equation (3) from equation (4)

$$\Rightarrow \quad \frac{18}{x} - \frac{16}{x} = \frac{3}{14} - \frac{1}{5}$$

$$\Rightarrow \quad \frac{2}{x} = \frac{1}{70} \quad \Rightarrow \quad x = 140$$

Substituting $x = 140$ in equation (2), we get

$$\frac{6}{140} + \frac{8}{y} = \frac{1}{14} \quad \Rightarrow \quad \frac{8}{y} = \frac{1}{14} - \frac{6}{140} \quad \Rightarrow \quad \frac{8}{y} = \frac{4}{140}$$

$$\Rightarrow \quad y = 280$$

Hence, time taken by one man alone to finish the work is 140 days.

And time taken by one woman alone to finish the work is 280 days.

29. Suppose point of intersection of chord QR and PC is M. Let tangent PQ be x and PM be y.

$$\therefore \quad QM = \frac{QR}{2} = \frac{16}{2} \text{ cm} = 8 \text{ cm} \quad \text{...(1)}$$

[PC is perpendicular bisector of QR]

In ΔQCM, $\quad CM^2 = QC^2 - QM^2 \quad$ [By Pythagoras theorem]

$$\Rightarrow \quad CM = \sqrt{10^2 - 8^2} \text{ cm} = 6 \text{ cm} \quad \text{[Using (1)]}$$

In right ΔPMQ, $\quad PQ^2 = MP^2 + QM^2$

$$\Rightarrow \quad x^2 = y^2 + 64 \text{ cm}^2 \quad \text{...(2)}$$

$$\angle PQC = 90^\circ$$

[Radius through the point of contact is perpendicular to the tangent]

In right ΔPQC, $\quad PC^2 = PQ^2 + QC^2$

$$\Rightarrow \quad (PM + CM)^2 = x^2 + 10^2 \text{ cm}^2$$

$$\Rightarrow \quad (y+6)^2 = x^2 + 100 \quad \Rightarrow \quad y^2 + 36 + 12y = y^2 + 64 + 100$$

[Using (2)]

$$\Rightarrow \quad 12y = 164 - 36 \quad \Rightarrow \quad y = 10.67$$

Putting value of y in equation (2)

$$x^2 = (10.67)^2 + 64 = 177.85$$

$$\Rightarrow \quad x = 13.33$$

Hence, PQ = 13.3 cm.

30. **Given:** $\sec\theta + \tan\theta = a$ ...(1)

Since $\sec^2\theta - \tan^2\theta = 1 \quad \Rightarrow \quad (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$$\Rightarrow \quad \sec\theta - \tan\theta = \frac{1}{a} \qquad \text{...(2)}$$

Adding equations (1) and (2), we get,

$$2\sec\theta = a + \frac{1}{a} \quad \Rightarrow \quad \sec\theta = \frac{a^2+1}{2a} \qquad \text{...(3)}$$

Subtracting equation (2) from (1), we get,

$$2\tan\theta = a - \frac{1}{a} \quad \Rightarrow \quad \tan\theta = \frac{a^2-1}{2a} \qquad \text{...(4)}$$

Now, $$\operatorname{cosec}\theta = \frac{\sec\theta}{\tan\theta} = \frac{\frac{a^2+1}{2a}}{\frac{a^2-1}{2a}} \qquad \text{[Using (3) and (4)]}$$

$$= \frac{a^2+1}{a^2-1}$$

31. Maximum class frequency 15 is of class 60–70 and mode is 67. So, modal class is 60–70.

$$\therefore \quad l = 60,\ h = 10,\ f_1 = 15,\ f_2 = 12,\ f_0 = x$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \quad 67 = 60 + \left(\frac{15 - x}{2 \times 15 - x - 12}\right) \times 10$$

$$\Rightarrow \quad 7(18 - x) = (15 - x) \times 10$$

$$\Rightarrow \quad 126 - 7x = 150 - 10x$$

$$\Rightarrow \quad 126 - 150 = -10x + 7x$$

$$\Rightarrow \quad -24 = -3x$$

$$\Rightarrow \quad x = 8$$

Hence, value of x is 8.

or

31. Here, $h = 10$. Let the assumed mean be $a = 55$.

Class Marks	*Mid value (x_i)*	*Number of students (f_i)*	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0 – 10	5	5	$\frac{5-55}{10} = -5$	– 25
10 – 20	15	9 – 5 = 4	$\frac{15-55}{10} = -4$	– 16
20 – 30	25	17 – 9 = 8	$\frac{25-55}{10} = -3$	– 24
30 – 40	35	29 – 17 = 12	$\frac{35-55}{10} = -2$	– 24
40 – 50	45	44 – 29 = 15	$\frac{45-55}{10} = -1$	– 15
50 – 60	55 = a	60 – 44 = 16	$\frac{55-55}{10} = 0$	0
60 – 70	65	70 – 60 = 10	$\frac{65-55}{10} = 1$	10
70 – 80	75	78 – 70 = 8	$\frac{75-55}{10} = 2$	16
80 – 90	85	83 – 78 = 5	$\frac{85-55}{10} = 3$	15
90 – 100	95	85 – 83 = 2	$\frac{95-55}{10} = 4$	8
		$\Sigma f_i = 85$		$\Sigma f_i u_i = -55$

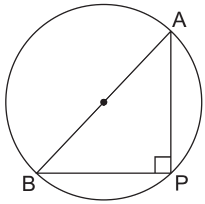
$$\text{Mean, } \bar{x} = a + h\bar{u} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 55 + 10 \times \left(\frac{-55}{85}\right) = 48.53$$

Hence, mean marks scored by the students is 48.53.

Section - D

32. Let P be the required location of pole such that its distance from gate B is x.



$\therefore$ $BP = x \Rightarrow AP = x + 7$

[Difference of the two distances is 7 m]

In right ΔAPB, $AB^2 = BP^2 + AP^2 \Rightarrow 17^2 = x^2 + (x+7)^2$

$\Rightarrow$ $289 = x^2 + x^2 + 49 + 14x$

$\Rightarrow$ $2x^2 + 14x - 240 = 0$

$\Rightarrow$ $x^2 + 7x - 120 = 0$

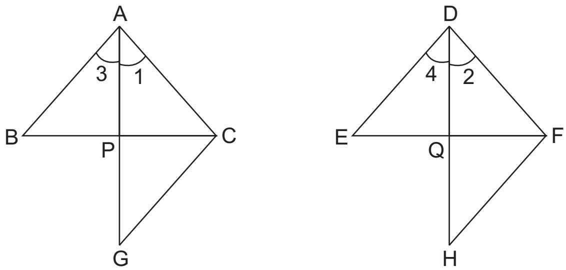
$\Rightarrow$ $(x + 15)(x - 8) = 0$

$\Rightarrow$ $x = -15$ or $x = 8$

Distance can't be negative.

Hence, pole is to be erected at 8 m from gates.

33. **Given:** ΔABC and ΔDEF in which AP and DQ are the medians respectively, such that $\frac{AB}{DE} = \frac{AC}{DF} = \frac{AP}{DQ}$



To prove: $\Delta ABC \sim \Delta DEF$

Proof: Produce AP to G so that PG = AP and join CG.

Similarly, DQ = QH and join FH.

In ΔAPB and ΔGPC, we have

$BP = CP$ [AP is median]

$AP = GP$

$\angle APB = \angle CPG$ [Vertically opposite angles]

$\therefore \quad \Delta APB \cong \Delta GPC$ [By SAS criterion of congruence]

$\therefore \quad AB = CQ$

[Corresponding sides of congruent triangle are equal]

Similarly, $\Delta DQE \cong \Delta HQF$ [BY SAS criterion of congruence]

$\therefore \quad DE = HF$

[Corresponding sides of congruent triangle are equal]

Now, $$\frac{AB}{DE} = \frac{AC}{DF} = \frac{AP}{DQ}$$

$$\Rightarrow \quad \frac{GC}{HF} = \frac{AC}{DF} = \frac{AP}{DQ}$$

$$\Rightarrow \quad \frac{GC}{HF} = \frac{AC}{DF} = \frac{2AP}{2DQ} \quad \Rightarrow \quad \frac{GC}{HF} = \frac{AC}{DF} = \frac{AG}{DH}$$

$\therefore \quad \Delta AGC \sim \Delta DHF$ [By SSS criterion of similarity]

$\therefore \quad \angle 1 = \angle 2$ [Corresponding angles of similar triangles]

Similarly, $\angle 3 = \angle 4$

$\Rightarrow \quad \angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow \quad \angle A = \angle D$

In ΔABC and ΔDEF

$$\angle A = \angle D$$

$$\frac{AB}{DE} = \frac{AC}{DF}$$

$\therefore \quad \Delta ABC \sim \Delta DEF$ [By SAS criterion of similarity]

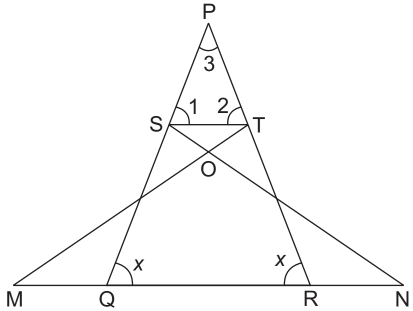
or

$\Delta NSQ \cong \Delta MTR$ [Given]

$\therefore \quad \angle SQN = \angle TRM$

[Corresponding angles of congruent triangles]

$\Rightarrow \quad \angle PQR = \angle PRQ = x$ ...(1)



In ΔPQR

$$\angle PQR + \angle PRQ + \angle QPR = 180°$$

$$x + x + \angle 3 = 180°$$

$$2x = 180° - \angle 3 \Rightarrow x = \frac{180° - \angle 3}{2} \quad ...(2)$$

In ΔPST

$$\angle 1 + \angle 2 + \angle 3 = 180°$$

$$2\angle 1 = 180° - \angle 3 \Rightarrow \angle 1 = \frac{180° - \angle 3}{2} \quad ...(3)$$

From (1), (2) and (3)

$$\angle 1 = x$$

$\therefore$ $$\angle 1 = \angle 2 = x = \angle PQR = \angle PRQ$$

In ΔPTS and ΔPRQ

$$\angle 1 = \angle PQR \quad \text{[Proved above]}$$

$$\angle 2 = \angle PRQ$$

$$\angle SPT = \angle QPR \quad \text{[Common]}$$

Hence, $$\Delta PTS \sim \Delta PRQ \quad \text{[By AAA criterion of similarity]}$$

34. Volume of object = Volume of cone + Volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \left(\frac{1}{3} \times \pi \times 5^2 \times 5 + \frac{2}{3} \times \pi \times 5^3\right) \text{cm}^3$$

$$= \frac{1}{3}\pi \times 5^3 \times 3 \text{ cm}^3 = \pi(5)^3 \text{ cm}^3$$

Volume of object = Volume of water displaced in the cylinder

$$\pi(5)^3 \text{ cm}^3 = \pi R^2 H$$

[R is radius of cylinder and H is the increase in water level]

$\Rightarrow$ $$(5)^3 \text{ cm}^3 = R^2 \times 4 \text{ cm}^3$$

$\Rightarrow$ $$R = 5.1 \text{ cm}$$

Hence, diameter of container is 11.2 cm.

or

(*i*) Let r_1 be the radius of cylinder and h_1 be the height of cylinder. Then, radius of cone be r_2 and height of cone be h_2.

Then $$r_1 = \frac{7}{2} \text{ cm}, h_1 = 15 \text{ cm}, r_2 = 3 \text{ cm}, h_2 = 4 \text{ cm}.$$

$$\text{Slant height} = \sqrt{r_2^2 + h_2^2} = \sqrt{3^2 + 4^2} \text{ cm} = \sqrt{25} \text{ cm} = 5 \text{ cm}$$

$\therefore$ $$l = 5 \text{ cm}$$

Surface area of remaining solid

$$= 2\pi r_1 h_1 + 2\pi r_1^2 - 2\pi r_2^2 + 2\pi r_2 l$$

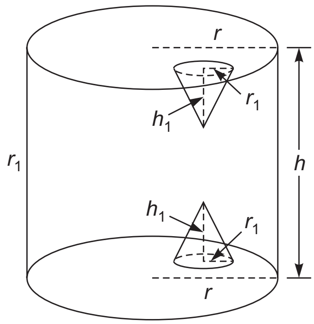
$$= 2 \times \pi(r_1 h_1 + r_1^2 - r_2^2 + r_2 l)$$

$$= 2\times\frac{22}{7}\left[\frac{7}{2}\times 15+\frac{49}{4}-9+15\right] \text{ cm}^2$$

$$= 444.7 \text{ cm}^2 \text{ (approx.)}$$

(*ii*) Let r and h be respectively the radius and the height of the solid cylinder and let r_1 and h_1 be the radius of the base and the height respectively of each of two identical conical holes at the two ends of the cylinder so that

$$r = \frac{7}{2} \text{ cm}, h = 15 \text{ cm}, r_1 = 3 \text{ cm and } h_1 = 4 \text{ cm.}$$



Now, volume of the cylinder

$$= \pi r^2 h$$

$$= \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 15 \text{ cm}^3$$

$$= 577.5 \text{ cm}^3 \quad ...(1)$$

Sum of the volumes of two identical conical holes at the two ends of the cylinder

$$= \frac{2}{3}\times \pi r_1^2 h_1$$

$$= \frac{2}{3}\times\frac{22}{7}\times 3\times 3\times 4 \text{ cm}^3$$

$$= 75.43 \text{ cm}^3 \quad ...(2)$$

∴ Required volume of the cylinder excluding the two conical holes

$$= (577.5 - 75.43) \text{ cm}^3 \quad \text{[From (1) and (2)]}$$

$$= 502.07 \text{ cm}^3$$

35. Here, $h = 3$. Let the assumed mean be $a = 11.5$.

Class	*Mid-value x_i*	*Frequency f_i*	*Cumulative frequency*	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
1 – 4	2.5	6	6	– 3	– 18
4 – 7	5.5	30	36	– 2	– 60
7 – 10	8.5	40	76	– 1	– 40
10 – 13	11.5 = a	16	92	0	0
13 – 16	14.5	4	96	1	4
16 – 19	17.5	4	100	2	8
		$\Sigma f_i = 100$			$\Sigma f_i u_i = -106$

$$\text{Mean, } \bar{x} = a + h \times \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) = 11.5 + 3 \times \left(\frac{-106}{100}\right) = 8.32$$

Now, $n = 100 \Rightarrow \frac{n}{2} = 50$

Cumulative frequency greater than 50 is 76 and corresponding class is 7 – 10.
Thus, median class is 7 – 10.

$$l = 7, h = 3, f = 40, cf = 36, \frac{n}{2} = 50$$

$$\text{Median} = l + h \times \frac{\left(\frac{n}{2} - cf\right)}{f}$$

$$= 7 + 3 \times \frac{(50 - 36)}{40}$$

$$= 7 + \frac{21}{20} = 8.05$$

$$\text{Mode} = 3\text{ Median} - 2\text{ Mean}$$

$$= 3 \times 8.05 - 2 \times 8.32 = 24.15 - 16.64 = 7.51$$

Hence, the mean number of alphabets in the names is 8.32, median is 8.05 and modal size is 7.51.

Section - E

36. (*i*) Number of passengers in first coach, $a_1 = 115$

Number of passengers in second coach, $a_2 = 130$

Difference of passengers between 2 coaches, $d = 130 - 115 = 15$

Number of passengers in tenth coach,

$$a_n = a_1 + (n - 1)d$$

$$\Rightarrow \quad a_{10} = 115 + (10 - 1) \times 15$$

$\Rightarrow \qquad a_{10} = 115 + 9 \times 15 \quad \Rightarrow \quad a_{10} = 250$

Hence, 250 passengers are seated in the tenth coach.

(*ii*) Passengers in fifth coach,

$$a_5 = 115 + (5 - 1) \times 15$$
$$= 115 + 4 \times 15 = 175$$

Extra passengers in tenth coach than fifth coach

$$= a_{10} - a_5$$
$$= 250 - 175 = 75$$

(*iii*) Collection of fare in first coach, $f_1 = 115 \times ₹\,500 = ₹\,57{,}500$

Collection of fare in second coach, $f_2 = 130 \times ₹\,500 = ₹\,65{,}000$

Difference of fare in 2 consecutive coaches

$$= f_2 - f_1$$
$$= ₹\,65{,}000 - ₹\,57{,}500 = ₹\,7500$$

or

Total passengers seated in train,

$$S_n = \frac{n}{2}\,[2a_1 + (n-1)d]$$

$$S_{10} = \frac{10}{2}\,[2 \times 115 + (10-1) \times 15]$$

$$S_{10} = 5[230 + 9 \times 15] \quad \Rightarrow \quad S_{10} = 5 \times [230 + 135]$$

$\Rightarrow \qquad S_{10} = 1825$

Hence, total 1825 passengers are seated in the train.

37. (*i*) Lines are along length. So, it will be along *x*-axis.

Potatoes are placed along breadth. So, it will be along *y*-axis. $\frac{1}{10}$th of breadth

$$= \frac{1}{10} \times 200 = 20$$

Hence, coordinates of Raju's flag is (5, 20).

(*ii*) Similarly $\frac{1}{8}$th of breadth $= \frac{1}{8} \times 200 = 25$

Hence, coordinates of Sanju's flag is (11, 25).

(*iii*) Coordinates of green flag = mid-point of line joining blue and yellow flag.

By mid-point formula,

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}$$

$\Rightarrow \qquad x = \frac{5 + 11}{2} \quad y = \frac{25 + 20}{2}$

$\Rightarrow \qquad x = 8, \quad y = \frac{45}{2}$

Hence, coordinates of green flag is $\left(8, \frac{45}{2}\right)$.

or

By distance formula.

Distance between blue and yellow flag

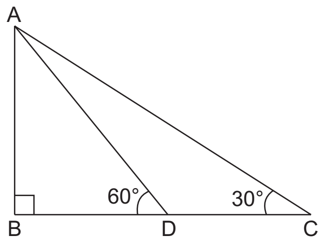
$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(11 - 5)^2 + (25 - 20)^2}$$

$$= \sqrt{6^2 + 5^2} = \sqrt{36 + 25} = \sqrt{61} \text{ units}$$

38. (*i*) Let AB be height of temple.

D is point on road. DC is the distance moved.



$$\text{DC} = 40 \text{ m} \qquad \text{[Given]}$$

In right ΔABD, we have

$$\tan 60° = \frac{\text{AB}}{\text{BD}}$$

$$\Rightarrow \quad \sqrt{3} = \frac{\text{AB}}{\text{BD}}$$

$$\Rightarrow \quad \text{BD} = \frac{\text{AB}}{\sqrt{3}} \qquad \text{...(1)}$$

In right ΔABC, we have

$$\tan 30° = \frac{\text{AB}}{\text{BC}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{\text{AB}}{\text{BD} + 40 \text{ m}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{\text{AB}}{\frac{\text{AB}}{\sqrt{3}} + 40 \text{ m}} \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{\text{AB}\sqrt{3}}{\text{AB} + 40\sqrt{3} \text{ m}}$$

$\Rightarrow \quad 3AB = AB + 40\sqrt{3}$ m $\quad \Rightarrow \quad 2AB = 40\sqrt{3}$ m

$\Rightarrow \quad AB = 20\sqrt{3}$ m

Hence, the height of the temple is $20\sqrt{3}$ m.

(*ii*) Area of $\Delta ABD = \frac{1}{2} \times AB \times BD$

$$= \frac{1}{2} \times 20\sqrt{3} \times 20 \text{ m}^2 = 200\sqrt{3} \text{ m}^2$$

Hence, the area of triangle formed is $200\sqrt{3}$ m^2.

(*iii*) Distance between temple and final point = BC

$$BC = BD + 40 \text{ m}$$

As, $\quad BD = \frac{AB}{\sqrt{3}} = \frac{20\sqrt{3} \text{ m}}{\sqrt{3}} = 20$ m

$\therefore \quad BC = (20 + 40)$ m $= 60$ m

Hence, distance from base of temple to final point of observation is 60 m.

or

Distance between top of temple and final point = AC

As, $\quad \sin 30° = \frac{AB}{AC}$

$\Rightarrow \quad \frac{1}{2} = \frac{20\sqrt{3} \text{ m}}{AC} \quad \Rightarrow \quad AC = 40\sqrt{3}$ m

Hence, distance from top of temple to final point of observation is $40\sqrt{3}$ m.

Sample Question Paper

Basic (Code 041)

ANSWERS

Section - A

1. (*d*)	2. (*a*)	3. (*c*)
4. (*c*)	5. (*b*)	6. (*c*)
7. (*d*)	8. (*a*)	9. (*c*)
10. (*d*)	11. (*a*)	12. (*b*)
13. (*d*)	14. (*d*)	15. (*c*)
16. (*a*)	17. (*b*)	18. (*a*)
19. (*d*)	20. (*b*)	

Section - B

21. Let us assume on the contrary that $\sqrt{3}$ is a rational number and its simplest form is $\frac{a}{b}$, where a and b are integers having no common factor other than 1 and $b \neq 0$.

$\therefore \quad \sqrt{3} = \frac{a}{b}$

$\Rightarrow \quad 3 = \frac{a^2}{b^2}$ [Squaring both sides]

$\Rightarrow \quad 3b^2 = a^2$ ...(1)

$\therefore$ a^2 is divisible by 3 [$\because$ $3b^2$ is divisible by 3]

$\Rightarrow$ a is divisible by 3 [$\because$ 3 is prime and divides a^2 $\Rightarrow$ 3 divides a]

Let $a = 3c$ for some integer c.

Substituting $a = 3c$ in (1), we get

$\Rightarrow \quad 3b^2 = (3c)^2 \Rightarrow 3b^2 = 9c^2 \Rightarrow b^2 = 3c^2$

$\Rightarrow$ b^2 is divisible by 3 [$\because$ $3c^2$ is divisible by 3]

$\Rightarrow$ b is divisible by 3 [$\because$ 3 is prime and divides b^2 $\Rightarrow$ 3 divides b]

Since, a and b are both divisible by 3, $\therefore$ 3 is a common factor of a and b.

But this contradicts the fact a and b have no common factor other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational.

Hence, $\sqrt{3}$ is an irrational number.

22. $$\tan (A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan (A - B) = \tan 30°$$

$$\Rightarrow \quad A - B = 30° \quad ...(1)$$

$$\sin (A + B) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \sin (A + B) = \sin 60°$$

$$\Rightarrow \quad A + B = 60° \quad ...(2)$$

Adding (1) and (2), we get

$$\Rightarrow \quad 2A = 90°$$

$$\Rightarrow \quad A = 45°$$

Substituting A = 45° in (1), we get

$$\therefore \quad B = 60° - A \quad \Rightarrow \quad B = 60° - 45° = 15°$$

or

$$\cos \theta + \sin \theta = \sqrt{2} \cos \theta$$

$$\Rightarrow \quad \sin \theta = \sqrt{2} \cos \theta - \cos \theta$$

$$\Rightarrow \quad \sin \theta = (\sqrt{2} - 1) \cos \theta$$

$$\Rightarrow \quad \cos \theta = \frac{\sin \theta}{\sqrt{2} - 1}$$

$$\Rightarrow \quad \cos \theta = \frac{\sin \theta}{(\sqrt{2} - 1)} \times \frac{(\sqrt{2} + 1)}{(\sqrt{2} + 1)}$$

$$\Rightarrow \quad \cos \theta = \frac{\sin \theta (\sqrt{2} + 1)}{2 - 1}$$

$$\Rightarrow \quad \cos \theta = \sqrt{2} \sin \theta + \sin \theta$$

$$\Rightarrow \quad \cos \theta - \sin \theta = \sqrt{2} \sin \theta$$ **Hence, proved.**

23. Distance covered is 1 revolution of wheel

$$= 2\pi r$$

$$= 2 \times \frac{22}{7} \times \frac{70}{2} = 220 \text{ cm}$$

According to the question, required speed of the bicycle

$$= 19.8 \text{ km/hr} = 5.5 \text{ m/s}$$

Distance covered by the wheel in 10 seconds

$$= \text{Speed} \times \text{Time}$$
$$= 5.5 \times 10 = 55 \text{ m}$$

Distance covered by the wheel in 10 seconds

$$= \text{Number of revolutions} \times \text{Distance covered in 1 revolution}$$

$$\Rightarrow \quad 55 = \text{Number of revolutions} \times \frac{220}{100} \qquad \left[1 \text{ cm} = \frac{1}{100} \text{ m}\right]$$

$$\Rightarrow \quad \text{Number of revolutions} = \frac{55}{220} \times 100 = 25$$

or

Radius of sector, $r = 60$ cm

Angle of major sector, $\theta = 360° - 90° = 270°$

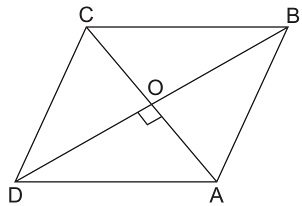
Perimeter of table top $=$ Perimeter of major sector $+ 2r$

$$= \frac{\theta}{360°} \times 2\pi r + 2r$$
$$= \frac{270°}{360°} \times 2 \times \frac{22}{7} \times 60 + 2 \times 60$$
$$= 282.85 + 120 = 402.85 \text{ cm}$$

24. Let ABCD be the rhombus, whose diagonals AC and BD bisect at O.

Diagonals of rhombus bisect each other at right angle.

Let AC = 50 cm and BD = 120 cm, $\angle AOB = 90°$



So, $AO = \frac{1}{2}AC$ and $OB = \frac{1}{2}BD$

$\Rightarrow \quad AO = \frac{1}{2} \times 50$ and $OB = \frac{1}{2} \times 120$

$\therefore \quad AO = 25$ cm, $OB = 60$ cm

In right ΔAOB, we have

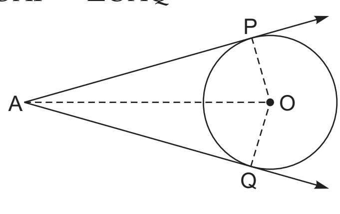
$$AB^2 = AO^2 + OB^2 \quad \Rightarrow \quad AB^2 = 25^2 + 60^2 = 4225$$

$$\Rightarrow \quad AB = 65$$

Hence, side of the rhombus is 65 cm.

25. **Given:** A circle with centre O and a point A outside it. AP and AQ are tangents drawn to the circle from point A.

To prove: $\angle OAP = \angle OAQ$



Proof: In ΔOAP and ΔOAQ, we have

$$OP = OQ \quad \text{[Radii of the same circle]}$$
$$AP = AQ \quad \text{[Tangents from an external point are equal]}$$
$$OA = OA \quad \text{[Common]}$$
$$\Delta OAP \cong \Delta OAQ \quad \text{[By SSS congruence]}$$
$$\therefore \quad \angle OAP = \angle OAQ \quad \text{[CPCT]}$$

Section - C

26. Given that $x + yp^{1/3} + zp^{2/3} = 0$ ...(1)

On multiplying both sides by $p^{1/3}$, we get

$$xp^{1/3} + yp^{2/3} + zp = 0 \quad ...(2)$$

Multiply (1) by y and (2) by z and subtracting, we get

$\Rightarrow \quad (xy + y^2p^{1/3} + yzp^{2/3}) - (xzp^{1/3} + yzp^{2/3} + z^2p) = 0$

$\Rightarrow \quad (y^2 - xz)p^{1/3} + xy - z^2p = 0$

$\Rightarrow \quad y^2 - xz = 0 \quad \text{and} \quad xy - z^2p = 0 \quad [\because p^{1/3} \text{ is irrational}]$

$\Rightarrow \quad y^2 = xz \quad \text{and} \quad xy = z^2p$

$\Rightarrow \quad y^2 = xz \quad \text{and} \quad x^2y^2 = z^4p^2$

$\Rightarrow \quad x^2(xz) = z^4p^2$

$\Rightarrow \quad x^3z - z^4p^2 = 0 \quad \Rightarrow \quad z(x^3 - z^3p^2) = 0$

$\Rightarrow \quad x^3 - z^3p^2 = 0 \quad \text{or} \quad z = 0$

$\Rightarrow \quad p^2 = \dfrac{x^3}{z^3}$

$\Rightarrow \quad p^{2/3} = \dfrac{x}{z}$

This is not possible as $p^{1/3}$ is irrational and $\dfrac{x}{z}$ is rational.

$\therefore \quad x^3 - p^2z^3 \neq 0$

Hence, $\quad z = 0$

Putting $z = 0$ in $y^2 - xz = 0$, we get $y = 0$

Putting $y = 0$ and $z = 0$ in (1)

$\Rightarrow \quad x = 0$

Hence, $\quad x = y = z = 0.$

27. Sum of roots (S) = –1

Product of roots (P) = –6

Required quadratic polynomial

$$x^2 - Sx + P$$
$$= x^2 - (-1)x + (-6)$$
$$= x^2 + x - 6$$

28. Let 1 man can finish the work in x days and 1 woman can finish it in y days.

1 man's 1 day's work = $\frac{1}{x}$

1 woman's 1 day's work = $\frac{1}{y}$

8 men and 12 women can finish the work in 10 days

$\Rightarrow$ (8 men's 1 day's work) + (12 women's 1 day's work) = $\frac{1}{10}$

$$\Rightarrow \quad \frac{8}{x} + \frac{12}{y} = \frac{1}{10} \quad ...(1)$$

6 men and 8 women can finish the work in 14 days

$\Rightarrow$ (6 men's 1 day's work) + (8 women's 1 day's work) = $\frac{1}{14}$

$$\Rightarrow \quad \frac{6}{x} + \frac{8}{y} = \frac{1}{14} \quad ...(2)$$

Multiply equation (1) by 2 and equation (2) by 3

$$\Rightarrow \quad \frac{16}{x} + \frac{24}{y} = \frac{1}{5} \quad ...(3)$$

$$\Rightarrow \quad \frac{18}{x} + \frac{24}{y} = \frac{3}{14} \quad ...(4)$$

Subtracting (3) from (4)

$$\Rightarrow \quad \frac{18}{x} - \frac{16}{x} = \frac{3}{14} - \frac{1}{5}$$

$$\Rightarrow \quad \frac{2}{x} = \frac{1}{70} \quad \Rightarrow \quad x = 140$$

From equation (2)

$$\frac{6}{140} + \frac{8}{y} = \frac{1}{14}$$

$$\Rightarrow \quad \frac{8}{y} = \frac{1}{14} - \frac{6}{140}$$

$$\Rightarrow \quad \frac{8}{y} = \frac{4}{140}$$

$$\Rightarrow \quad y = 280$$

Hence, time taken by man alone to finish the work is 140 days, and time taken by woman alone to finish the work is 280 days.

or

$$\frac{x}{3} + \frac{y}{4} = 6 \quad ...(1)$$

$$\frac{x}{6} + \frac{y}{2} = 6 \quad ...(2)$$

Dividing equation (1) by 2, we get

$$\Rightarrow \quad \frac{x}{6}+\frac{y}{8} = 3 \quad \ldots(3)$$

Subtracting equation (3) from equation (2), we get

$$\frac{x}{6}+\frac{y}{2}-\frac{x}{6}-\frac{y}{8} = 6-3$$

$$\Rightarrow \quad \frac{y}{2}-\frac{y}{8} = 3$$

$$\Rightarrow \quad \frac{3y}{8} = 3$$

$$\Rightarrow \quad y = 8$$

Substituting $y = 8$ in equation (1), we get

$$\frac{x}{3}+\frac{8}{4} = 6$$

$$\Rightarrow \quad \frac{x}{3}+2 = 6$$

$$\Rightarrow \quad \frac{x}{3} = 4 \quad \Rightarrow \quad x = 12$$

$$\therefore \quad 3y-2x = 3\times 8-2\times 12 = 0$$

and $\frac{x}{y}+\frac{1}{2}$

$$= \frac{12}{8}+\frac{1}{2}$$

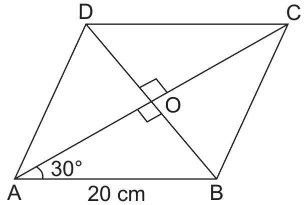
$$= \frac{3}{2}+\frac{1}{2}$$

$$= 2$$

29. In the given diagonals AC and BD bisect each other at right angles at O.

$$\Delta AOB \cong \Delta AOD \quad \text{[By RHS congruence]}$$

$$\therefore \quad \angle BAO = \angle DAO = \frac{1}{2}\angle DAB = \frac{1}{2}\times 60^\circ = 30^\circ \quad \text{[CPCT]}$$



In right ΔAOB,

$$\sin 30^\circ = \frac{OB}{AB} \quad \Rightarrow \quad \frac{1}{2}=\frac{OB}{20} \quad \Rightarrow \quad OB = 10 \text{ cm}$$

$$BD = 2OB \quad \Rightarrow \quad BD = 2\times 10 = 20 \text{ cm}$$

$$\cos 30° = \frac{AO}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AO}{20} \Rightarrow AO = 10\sqrt{3} \text{ cm}$$

$$AC = 2AO \Rightarrow AC = 2 \times 10\sqrt{3} = 20\sqrt{3} \text{ cm}$$

Hence, length of diagonals are 20 cm and $20\sqrt{3}$ cm.

30. Class size = Difference between any two consecutive mid values = 25 – 15 = 10.
Mid values 15 corresponds to class (15 – 5) – (15 + 5), *i.e.* 10 – 20, and so on.
Thus, cumulative frequency table for the given data is:

Class Interval	*Frequency* (f_i)	*Cumulative Frequency* (cf)
10 – 20	4	4
20 – 30	28	32
30 – 40	15	47
40 – 50	20	67
50 – 60	17	84
60 – 70	16	100
Total	$n = \Sigma f_i = 100$	

Here, $n = \Sigma f_i = 100 \Rightarrow \frac{n}{2} = \frac{100}{2} = 50.$

The cumulative frequency just greater than 50 is 67 and the corresponding class is 40 – 50. So, the median class is 40 – 50.

$\therefore$ $l = 40, cf = 47, f = 20$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 40 + \left(\frac{50 - 47}{20}\right) \times 10 = 40 + \frac{3}{20} \times 10$$

$$= 40 + 1.5$$

$$= 41.5$$

Hence, the median is 41.5.

31. $\angle PQO = 90°$ ...(1) [The tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\angle POQ + \angle POR = 180°$ [Straight angle]

$\Rightarrow \angle POQ + 130° = 180°$

$$\Rightarrow \quad \angle POQ = 50°$$

In ΔOPQ, we have

$$\angle OPQ + \angle PQO + \angle POQ = 180° \quad \text{[Sum of angles of a triangle is 180°]}$$

$$\Rightarrow \quad \angle 1 + 90° + 50° = 180° \quad \text{[Using (1) and (2)]}$$

$$\Rightarrow \quad \angle 1 = 40° \quad ...(3)$$

$$\angle RST = \frac{1}{2} \angle TOR$$

[Angle subtended by an arc of a circle at the center is twice the angle subtended by it at any point on remaining part of the circle]

$$\Rightarrow \quad \angle 2 = \frac{1}{2} \angle POR$$

$$\Rightarrow \quad \angle 2 = \frac{1}{2} \times 130° = 65° \quad ...(4)$$

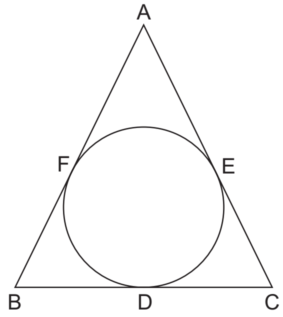
Adding (3) and (4), we get

$$\therefore \quad \angle 1 + \angle 2 = 40° + 65° = 105°$$

Hence, $\angle 1 + \angle 2 = 105°$

or

Since, tangents drawn from an external point to a circle are equal.



$$BF = BD,\ CD = CE \text{ and } AE = AF \quad ...(1)$$

$$\text{Semi-perimeter} = \frac{AB + BC + CA}{2} = s$$

$$\Rightarrow \quad AB + BC + CA = 2s$$

$$\Rightarrow \quad (AF + BF) + (BD + CD) + (CE + AE) = 2s$$

$$\Rightarrow \quad (AE + BD) + (BD + CE) + (CE + AE) = 2s \quad [\because BF = BD, CD = CE, AE = AF]$$

$$\Rightarrow \quad 2(AE + CE) + 2BD = 2s$$

$$\Rightarrow \quad 2AC + 2BD = 2s$$

$$\Rightarrow \quad AC + BD = s$$

$\Rightarrow \quad BD = s - AC$ **Hence, proved.**

Section - D

32. Let total number of birds be x.

Then, number of birds moving about in lotus plant = $\frac{x}{4}$.

The number of birds moving on a hill = $\frac{x}{9}+\frac{x}{4}+7\sqrt{x}$

Number of birds remain on trees = 56

$\therefore$ Number of birds moving in lotus plant + Birds moving on hill + Birds remain on trees = Total number of birds

$$\Rightarrow \quad \frac{x}{4}+\frac{x}{9}+\frac{x}{4}+7\sqrt{x}+56 = x$$

$$\Rightarrow \quad \left(\frac{x}{4}+\frac{x}{9}+\frac{x}{4}-x\right)+7\sqrt{x}+56 = 0$$

$$\Rightarrow \quad \left(\frac{9x+4x+9x-36x}{36}\right)+7\sqrt{x}+56 = 0$$

$$\Rightarrow \quad \frac{-14x}{36}+7\sqrt{x}+56 = 0$$

$$\Rightarrow \quad x-18\sqrt{x}-144 = 0 \qquad \left[\text{Multiply by } \frac{-36}{14}\right]$$

Let $\sqrt{x} = y$

$$\Rightarrow \quad y^2-18y-144 = 0$$

$$\Rightarrow \quad y^2-24y+6y-144 = 0 \quad \Rightarrow \quad y(y-24)+6(y-24) = 0$$

$$\Rightarrow \quad (y+6)(y-24) = 0$$

$$\Rightarrow \quad y = -6 \quad \text{or} \quad y = 24$$

$$\therefore \quad \sqrt{x} = -6 \quad \text{or} \quad \sqrt{x} = 24$$

Since, $\sqrt{x} = -6$ is not possible

$$\therefore \quad x = (24)^2 = 576$$

Hence, the total number of birds is 576, which is in accordance with problem.

or

The given equation is

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a) = 0$$

$$\Rightarrow \quad 3x^2-2(a+b+c)x+(ab+bc+ca) = 0$$

Comparing the given equation with $a_1x^2+b_1x+c_1 = 0$, we have

$$a_1 = 3,\ b_1 = -2(a+b+c),\ c_1 = ab+bc+ca$$

$$\begin{aligned} D &= b_1^2-4a_1c_1 \\ &= 4(a+b+c)^2-4\times 3(ab+bc+ca) \\ &= 4(a^2+b^2+c^2+2ab+2bc+2ca)-12(ab+bc+ca) \end{aligned}$$

$$= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 3ab - 3bc - 3ca)$$
$$= 4(a^2 + b^2 + c^2 - ab - bc - ca)$$
$$= 2(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$$
$$= 2[(a^2 + b^2 - 2ab) - (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ca)]$$
$$= 2[(a - b^2) + (b - c)^2 + (c - a)^2]$$
$$[\because \quad (a - b)^2 \geq 0, (b - c)^2 \geq 0, (c - a)^2 \geq 0]$$

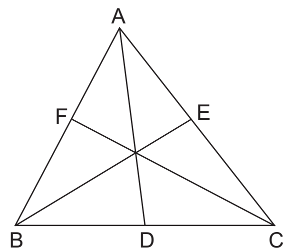
As, $D \geq 0$, roots of the given equation are real.

For equal roots

$$D = 0$$
$$\Rightarrow \quad (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$$
$$\Rightarrow \quad a - b = 0, b - c = 0, c - a = 0 \quad \Rightarrow \quad a = b = c$$
$$\Rightarrow \quad a = b, b = c, c = a$$

Hence, the roots are equal only when $a = b = c$.

33. **Given:** In ΔABC, AD, BE and CF are medians.



To prove: $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$

Proof: Since, in any triangle, sum of squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median bisecting it.

Therefore, we have,

$$AB^2 + AC^2 = 2(AD^2 + BD^2) \quad ...(1)$$
$$BC^2 + AB^2 = 2(BE^2 + CE^2) \quad ...(2)$$
and
$$CA^2 + BC^2 = 2(CF^2 + AF^2) \quad ...(3)$$

Adding the corresponding sides of equations (1), (2) and (3), we get

$$2(AB^2 + BC^2 + CA^2) = 2(AD^2 + BD^2 + BE^2 + CE^2 + CF^2 + AF^2)$$
$$\Rightarrow \quad AB^2 + BC^2 + AC^2 = AD^2 + BE^2 + CF^2 + \left(\frac{BC}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{AB}{2}\right)^2$$
$$[\because \text{ AD, BE and CF are medians } \therefore BD = \frac{1}{2}BC, CE = \frac{1}{2}AC \text{ and } AF = \frac{1}{2}AB]$$
$$\Rightarrow \quad AB^2 - \frac{AB^2}{4} + BC^2 - \frac{BC^2}{4} + CA^2 - \frac{CA^2}{4} = AD^2 + BE^2 + CF^2$$
$$\Rightarrow \quad \frac{3}{4}AB^2 + \frac{3}{4}BC^2 + \frac{3}{4}CA^2 = AD^2 + BE^2 + CF^2$$
$$\Rightarrow \quad 3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$$

34. Let R be the radius and H the height of the cylindrical tank.

Then radius (R) of tank $= \frac{10}{2}$ m = 5 m and H = 2 m

$\therefore$ Volume of cylindrical tank = $\pi R^2 H$

$$= \pi \times (5)^2 \times 2 \text{ m}^3 \qquad ...(1)$$

Rate of flow of water through the pipe

$$= 6 \text{ km/h} = \frac{6 \times 1000}{60} = 100 \text{ m/min.} \qquad ...(2)$$

Let r be the radius of the cross-section of the pipe

$$= \frac{20}{2} = 10 \text{ cm} = \frac{1}{10} \text{ m}$$

Area of cross-section of the pipe = $\pi r^2 = \pi\left(\frac{1}{10}\right)^2 \text{ m}^2$

Volume of water flowing through pipe in 1 minute

$$= \pi\left(\frac{1}{10}\right)^2 \times 100 \text{ m}^2 \qquad \text{[using (2)]} \qquad ...(3)$$

Suppose the tank gets filled in x minutes.

$$\begin{bmatrix}\text{Volume of water flowing through} \\ \text{pipe in } x \text{ minutes}\end{bmatrix} = \begin{bmatrix}\text{Volume of water} \\ \text{in fill tank}\end{bmatrix}$$

$$\Rightarrow \quad \pi\left(\frac{1}{10}\right)\left(\frac{1}{10}\right)(100) \times x = \pi(5)^2 (2)$$

$$\Rightarrow \quad \pi \times \left(\frac{1}{10}\right)^2 (100) \times x = \pi \times (5)^2 \times 2 \qquad \text{[Using (1) and (3)]}$$

$$\Rightarrow \quad x = 5 \times 5 \times 2 = 50$$

Hence, tank will get filled up in 50 minutes.

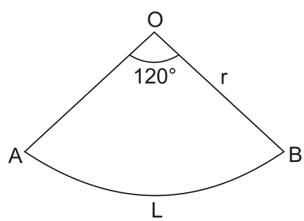
or

Let AOB represent the sector of the circle with radius r = 15 cm and central angle AOB = 120° (= θ say).

Then, length of arc AB = $\frac{\theta}{360} \times 2\pi r$

$$= \left(\frac{120}{360} \times 2 \times \frac{22}{7} \times 15\right) \text{cm} = \frac{220}{7} \text{ cm} \qquad ...(1)$$

When sector is rolled up, arc AB forms the circumference of base of the cone and radius (r = 15 cm) forms the slant height of cone.



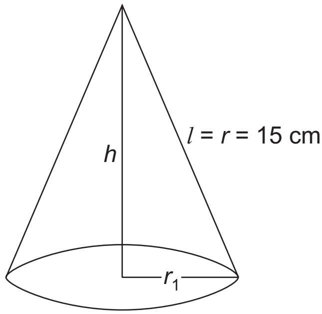
$$\therefore \quad l = 15 \text{ cm}$$

Let r_1 be the radius of cone and h its height.

$$\therefore \quad 2\pi r_1 = \frac{220}{7} \text{ cm} \qquad \text{[Using (1)]}$$

$$\Rightarrow \qquad r_1 = \frac{220}{7} \times \frac{1}{2} \times \frac{7}{22} = 5 \text{ cm}$$

$$h = \sqrt{l^2 - r_1^2} = \sqrt{15^2 - 5^2} = \sqrt{200} = 10\sqrt{2} \text{ cm}$$



$$\therefore \qquad \text{Volume of cone} = \frac{1}{3}\pi r_1^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 10\sqrt{2} = 370.38 \text{ cm}^3.$$

Hence, the volume of the cone is 370.38 cm^3 (approx.).

35.

Height	*Mid value* (x_i)	*Number of girls* (f_i)	*Cumulative frequency (cf)*	$f_i x_i$
120 – 130	125	2	2	250
130 – 140	135	8	10	1080
140 – 150	145	12	22	1740
150 – 160	155	20	42	3100
160 – 170	165	8	50	1320
Total	$n = \Sigma f_i = 50$			$\Sigma f_i x_i = 7490$

$$\textbf{Mean: } \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{7490}{50} = 149.8$$

$$\textbf{Median: } n = \Sigma f_i = 50, \text{ so } \frac{n}{2} = \frac{50}{2} = 25$$

Cumulative frequency just greater than 25 is 42 and the corresponding class is 150 – 160. So the median class is 150 – 160.

$$\therefore \qquad l = 150, cf = 22, f = 20, h = 10$$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 150 + \left(\frac{25 - 22}{20}\right) \times 10 = 150 + 1.5$$

$$= 151.5$$

Mode: Here the maximum frequency is 20 and class corresponding to it is 150 – 160. So modal class is 150 – 160.

$\therefore$ $l = 150, f_1 = 20, f_0 = 12, f_2 = 8, h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 150 + \left(\frac{20 - 12}{2 \times 20 - 12 - 8}\right) \times 10$$

$$= 150 + \frac{8}{20} \times 10$$

$$= 150 + 4 = 154$$

Hence, mean height of the girls is 149.8 cm, their median height is 151.5 cm and their modal height is 154 cm.

Section - E

36. Let the sale of smartphones in 1st year be a_1 and 2nd year be a_2.

Difference in sale of phones in 2nd year and 1st year is d.

Sale of smartphones in 3rd year, $a_3 = 10{,}000$

Sale of smartphones in 5th year, $a_5 = 14{,}000$

(*a*) Sale in 2nd year = a_2.

$\therefore$ $a_3 = a_1 + (3 - 1)d$ $[a_n = a_1 + (n - 1)d]$

$\Rightarrow$ $10000 = a_1 + 2d$ ...(1)

$a_5 = a_1 + 4d \Rightarrow 14000 = a_1 + 4d$ ...(2)

Subtracting equation (1) from equation (2), we get

$\Rightarrow$ $14000 - 10000 = a_1 + 4d - a_1 - 2d$

$\Rightarrow$ $4000 = 2d \Rightarrow d = \frac{4000}{2} = 2000$

$\therefore$ $10000 = a_1 + 2 \times 2000$

$\Rightarrow$ $a_1 = 10000 - 4000 = 6000$

$a_2 = a_1 + d = 6000 + 2000 = 8000$

Hence, the sale in 2nd year is 8000.

(*b*) Total sale in first 5 years = S_5

$$S_5 = \frac{5}{2}(6000 + 14000) \qquad \left[S_n = \frac{n}{2}(a_1 + a_n)\right]$$

$$= \frac{5}{2} \times 20000 = 50000$$

Hence, total sale of smartphones in first 5 years is 50,000.

or

Let the sale be a_n in n^{th} year.

$\therefore \quad a_n = a_1 + (n-1)d$

$\Rightarrow \quad 22000 = 6000 + (n-1) \times 2000$

$\Rightarrow \quad 22000 - 6000 = (n-1) \times 2000$

$\Rightarrow \quad 16000 = (n-1) \times 2000$

$\Rightarrow \quad 8 = n - 1 \quad \Rightarrow \quad n = 9$

Hence, sale would be 22000 after 9^{th} year.

(*c*) Let the total sale will be S_n in n^{th} years.

$\therefore \quad S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$\Rightarrow \quad 66000 = \frac{n}{2}[2 \times 6000 + (n-1)\, 2000]$

$\Rightarrow \quad 66000 = 6000n + (n^2 - n)1000$

$\Rightarrow \quad 66 = 6n + n^2 - n$

$\Rightarrow \quad n^2 + 5n - 66 = 0$

$\Rightarrow \quad n^2 + 11n - 6n - 66 = 0 \quad \Rightarrow \quad (n-6)(n+11) = 0$

$\Rightarrow \quad n = 6, -11$

As –11 can't be possible year.

Hence, total sale would be 66,000 after 6 years.

37. (*a*) Coordinates of C is (–5, –3), A is (2, 7), B is (7, –8)

By distance formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance between A and C $= \sqrt{(2+5)^2 + (7+3)^2}$

$= \sqrt{49 + 100} = \sqrt{149}$ unit

Distance between B and C $= \sqrt{(7+5)^2 + (-8+3)^2}$

$= \sqrt{144 + 25} = \sqrt{169} = 13$ unit

Hence, Dev travelled more distance.

(b) By mid-point formula coordinates of mid-point of BA is (x, y)

$$x = \frac{7+2}{2}, y = \frac{-8+7}{2} \qquad \left[x = \frac{x_1+x_2}{2}, y = \frac{y_1+y_2}{2}\right]$$

$$x = \frac{9}{2}, y = -\frac{1}{2}$$

Hence, coordinates of that place is $\left(\frac{9}{2}, -\frac{1}{2}\right)$

or

Area of triangle formed by points A, B and C is ABC

$$= \frac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}\,|2(-8-7) + 7(-3-7) + (-5)\,(7+8)|$$

$$= \frac{1}{2}\,|2(-15) + 7(-10) - 5 \times 15|$$

$$= \frac{1}{2}(-30 - 70 - 75)$$

$$= \frac{1}{2}(-175) = 87.5 \text{ sq units.}$$

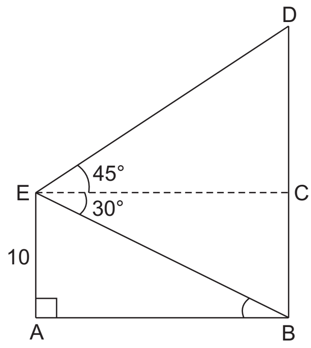
Hence, area of the triangle formed by A, B and C is 87.5 unit square.

(c) Distance between A and B = $\sqrt{(7-2)^2 + (-8-7)^2}$

$$= \sqrt{25 + 225}$$

$$= \sqrt{250} \text{ unit} = 5\sqrt{10} \text{ unit}$$

38. Let AE be height of the tree and BD is height of the tower.



AB is the distance between the tree and the tower.

Angle of elevation, ∠DEC = 45°

Angle of depression, ∠CEB = 30°

(i) ∠ABE = ∠CEB = 30° [Alternate angles]

In ΔAEB,

$$\tan 30° = \frac{AE}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{AB} \quad \Rightarrow \quad AB = 10\sqrt{3} \text{ m}$$

Therefore, distance between the tree and the tower = $10\sqrt{3}$ m

(ii) In ΔCDE,

$$\tan 45° = \frac{CD}{EC}$$

$$\Rightarrow \quad 1 = \frac{CD}{10\sqrt{3}} \qquad [EC = AB]$$

$$\Rightarrow \quad CD = 10\sqrt{3} \text{ m}$$

$$\therefore \quad BD = BC + CD = 10 + 10\sqrt{3} = 10\,(1 + \sqrt{3}) \text{ m}$$

Therefore, height of the tower = $10\,(1 + \sqrt{3})$ m

Area of trapezium ABDE = $\frac{1}{2}$ × [sum of lengths of parallel sides × distance between them]

$$= \frac{1}{2}\left[10 + 10(1 + \sqrt{3})\right] \times 10\sqrt{3} \text{ m}^2$$

$$= \left[20 + 10\sqrt{3}\right] \times 5\sqrt{3} = (100\sqrt{3} + 150) \text{ m}^2.$$

or

$$\sin 45^\circ = \frac{CD}{ED}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{10\sqrt{3} \text{ m}}{ED}$$

$$\Rightarrow \quad ED = 10\sqrt{6} \text{ m}$$

Perimeter of the trapezium = AB + BD + ED + AE

$$= (10\sqrt{3} + 10 + 10\sqrt{3} + 10\sqrt{6} + 10) \text{ m}$$

$$= (20 + 20\sqrt{3} + 10\sqrt{6}) \text{ m}$$

∴ Perimeter of the trapezium = $10(2 + 2\sqrt{3} + \sqrt{6})$ m.

Hence, perimeter of the trapezium is $10(2 + 2\sqrt{3} + \sqrt{6})$ m.

(*iii*) Height of the tower = $BC + CD = 10 + 10\sqrt{3}$

$$= 10(1 + \sqrt{3}) \text{ m}.$$

Hence, height of the tower = $10(1 + \sqrt{3})$ m.